

Optimizing transmon readout with dynamiqs, a library for GPU-accelerated differentiable quantum simulations

Ronan Gautier^{1,2,3}, Élie Genois¹, Pierre Guilmin^{2,3}, Adrien Bocquet², Alexandre Blais¹
Centre for Quantum Dynamics / 16th April 2024

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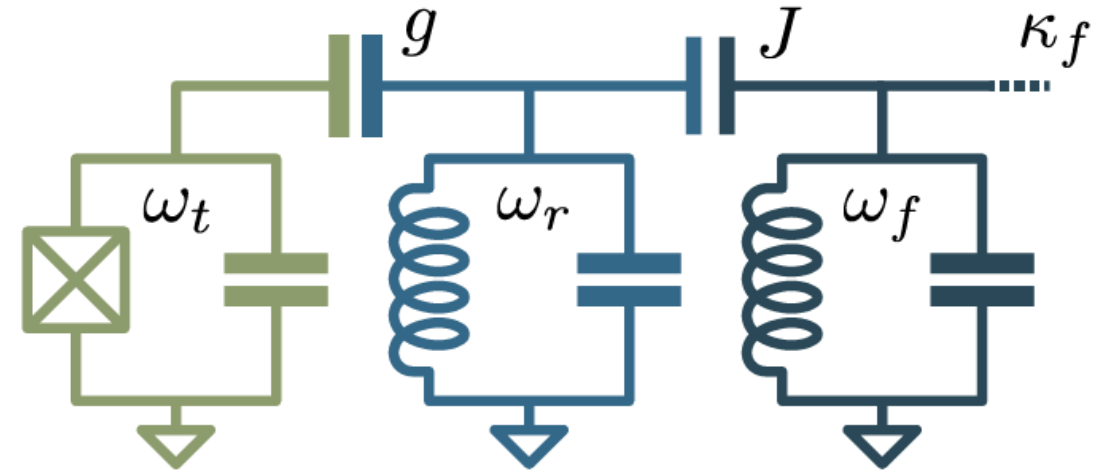
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Dispersive readout

We want to optimize the dispersive readout of a transmon.

$$H = 4E_C n_t^2 - E_J \cos(\phi_t) + \omega_r a^\dagger a + \omega_f f^\dagger f + i g n_t (a^\dagger - a) + J (f^\dagger a + a^\dagger f)$$

- Full cosine model, including Purcell filter
- MW drive on Purcell filter and/or transmon



Difficult numerical problem

- ~400 parameters (1ns bins x 100ns x 2 drives)
- Hilbert space size ~ 8000 (5 x 40 x 40)
- GHz dynamics
- Open quantum system

Quantum optimal control



Gradient-free methods

- CRAB (Doria, PRL 2010)
- Nelder-Mead (Egger, PRL 2014)
- Model-free RL (Sivak, PRX 2022)

↳ Do not scale to many parameters

Gradient-based methods

- GRAPE (Boutin, PRA 2016)
- Krotov (Koch, JP:CM 2016)

↳ Only state-to-state or gate optim.

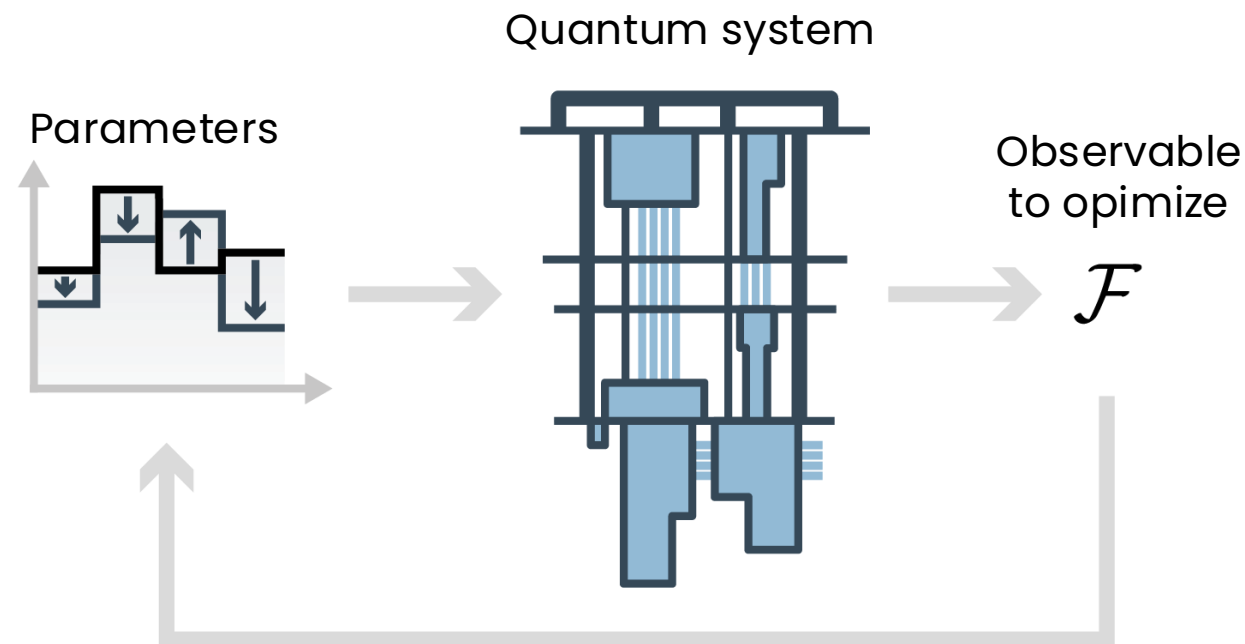
- Automatic differentiation (Leung, PRA 2017)

↳ $O(N_T \times N^2)$ memory \rightarrow 4 Terabytes !

- Adjoint state method



Low memory + any closed or open system + any parametrized problem + fast



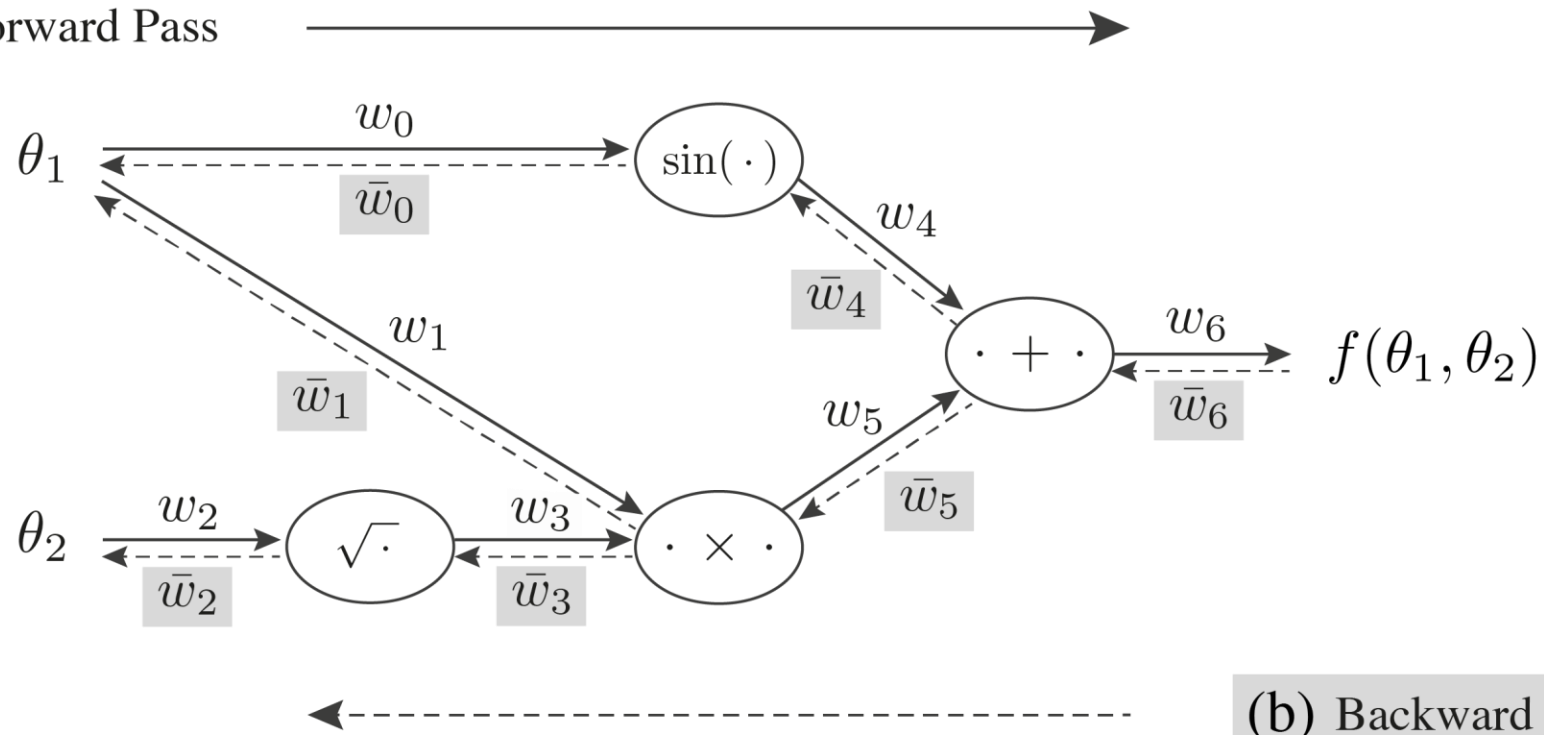
Primer on automatic differentiation

We want to differentiate the function

$$f(\theta_1, \theta_2) = \sin(\theta_1) + \theta_1 \sqrt{\theta_2}$$

Graph of operations

(a) Forward Pass



(b) Backward Pass

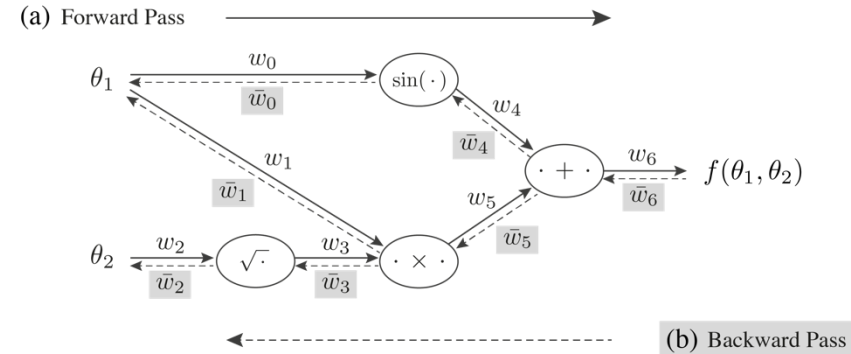
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Table of operations

Forward pass	Backward pass
$w_0 = \theta_1$	$\bar{w}_0 = \bar{w}_4 \frac{\partial w_4}{\partial w_0} = \bar{w}_4 \cos(w_0) = \cos(\theta_1)$
$w_1 = \theta_1$	$\bar{w}_1 = \bar{w}_5 \frac{\partial w_5}{\partial w_1} = \bar{w}_5 w_3 = \sqrt{\theta_2}$
$w_2 = \theta_2$	$\bar{w}_2 = \bar{w}_3 \frac{\partial w_3}{\partial w_2} = \bar{w}_3 / 2\sqrt{w_2} = \theta_1 / 2\sqrt{\theta_2}$
$w_3 = \sqrt{w_2} = \sqrt{\theta_2}$	$\bar{w}_3 = \bar{w}_5 \frac{\partial w_5}{\partial w_3} = \bar{w}_5 w_1 = \theta_1$
$w_4 = \sin w_0 = \sin \theta_1$	$\bar{w}_4 = \bar{w}_6 \frac{\partial w_6}{\partial w_4} = \bar{w}_6 \cdot 1 = 1$
$w_5 = w_1 w_3 = \theta_1 \sqrt{\theta_2}$	$\bar{w}_5 = \bar{w}_6 \frac{\partial w_6}{\partial w_5} = \bar{w}_6 \cdot 1 = 1$
$w_6 = w_4 + w_5 = \sin(\theta_1) + \theta_1 \sqrt{\theta_2}$	$\bar{w}_6 = 1$ (seed)



Differentiating through a matrix multiplication requires storing the original matrices!

Adjoint state method

- Parametrized master equation

$$\frac{d\rho}{dt} = \mathcal{L}\rho = -i[H, \rho] + \sum \mathcal{D}[L_k]\rho$$

$$\hookrightarrow H = H(\theta) \quad \hookrightarrow L_k = L_k(\theta)$$

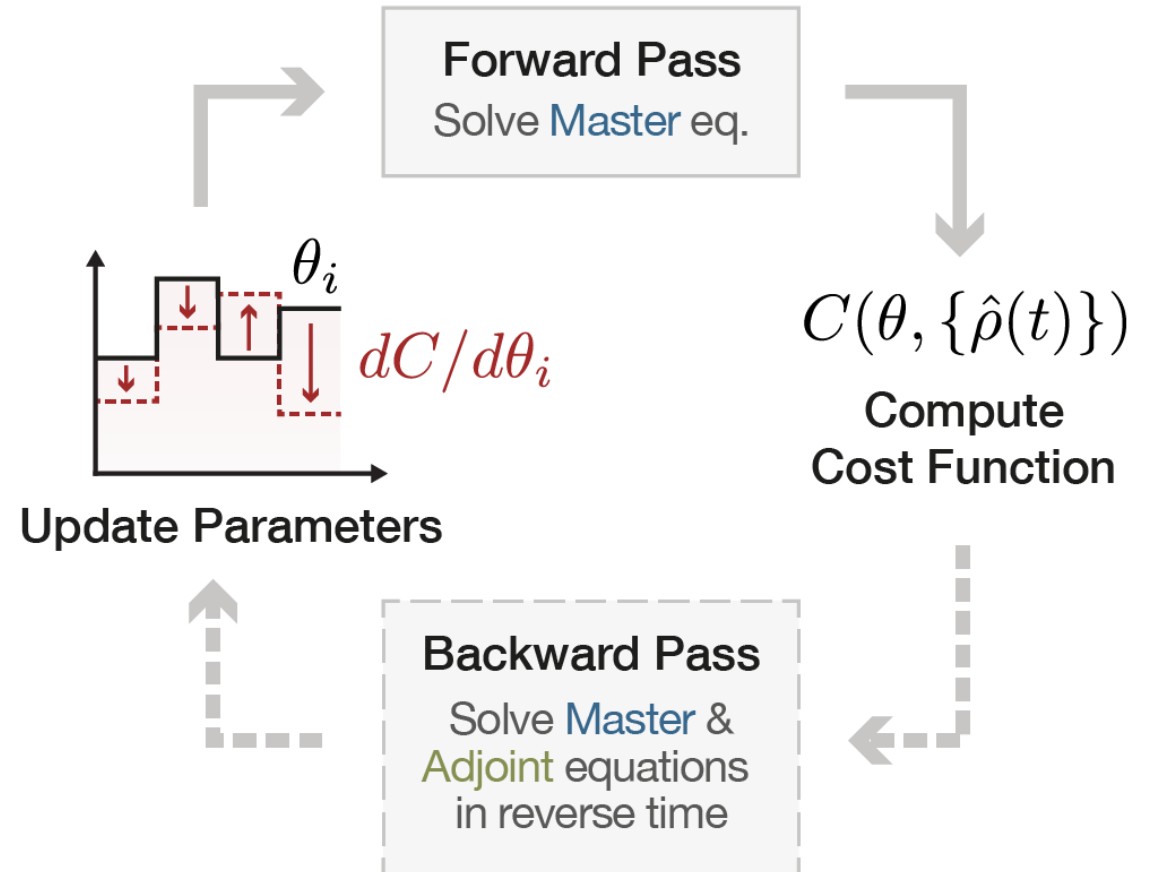
- Cost function $C = C(\theta, \rho(t_0), \dots, \rho(t_n))$

- Adjoint state $\phi(t) = dC/d\rho(t)$

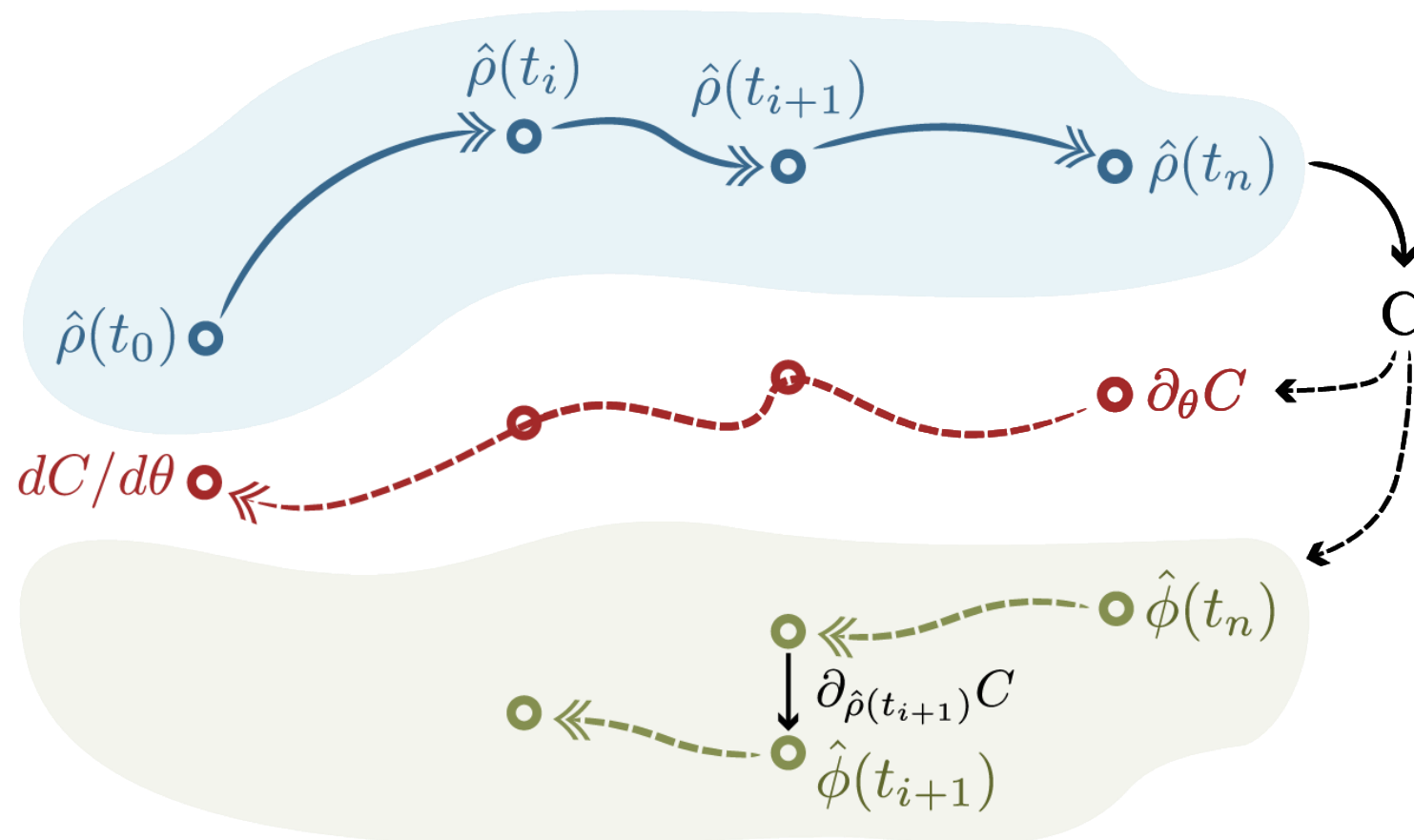
$$\frac{d\phi}{dt} = -\mathcal{L}^\dagger \phi = -i[H, \phi] - \sum \mathcal{D}^\dagger[L_k]\phi$$

- Explicit expression of gradient

$$\frac{dC}{d\theta} = \frac{\partial C}{\partial \theta} - \int_{t_0}^{t_n} \partial_\theta \text{Tr} [\phi^\dagger(t) \mathcal{L}(t, \theta) \rho(t)] dt$$



Reverse-time backpropagation

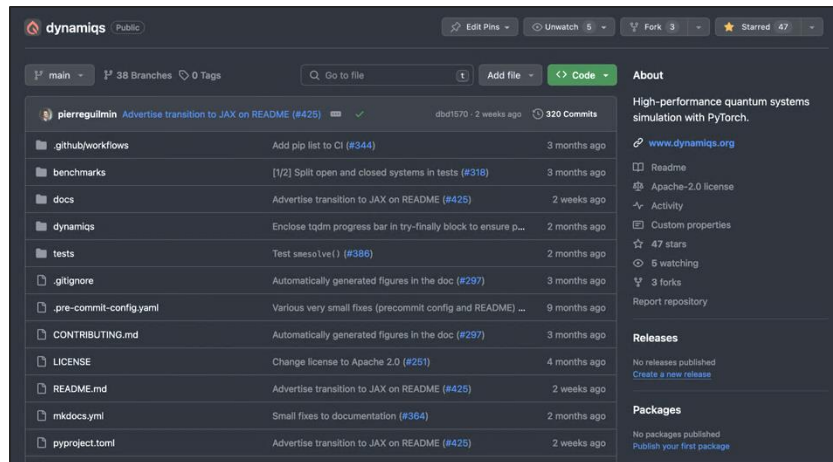


Memory: $\mathcal{O}(N^2)$ → 488 MB

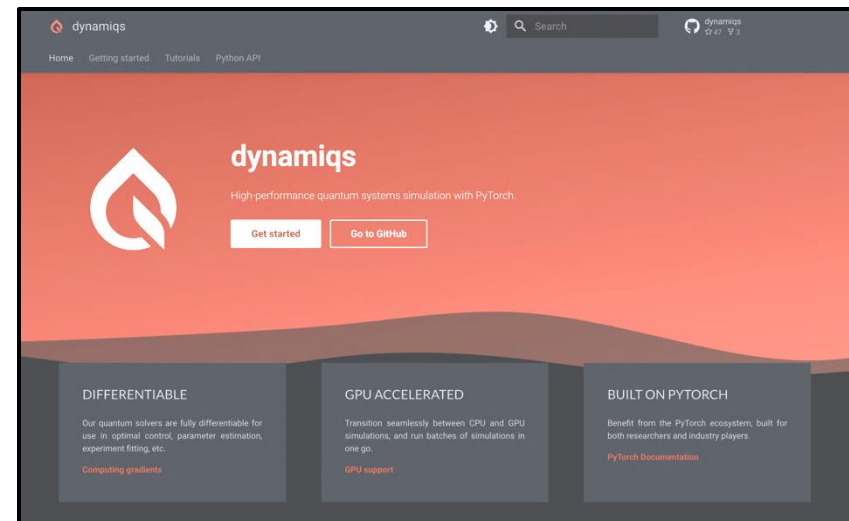
Optimal control with dynamiqs

- Open-source
- Closed, open and stochastic quantum systems
- End-to-end differentiable
- Works on GPUs → (very) fast simulations
- Batching
- QuTiP-like API
- ...

github.com/dynamiqs



www.dynamiqs.org

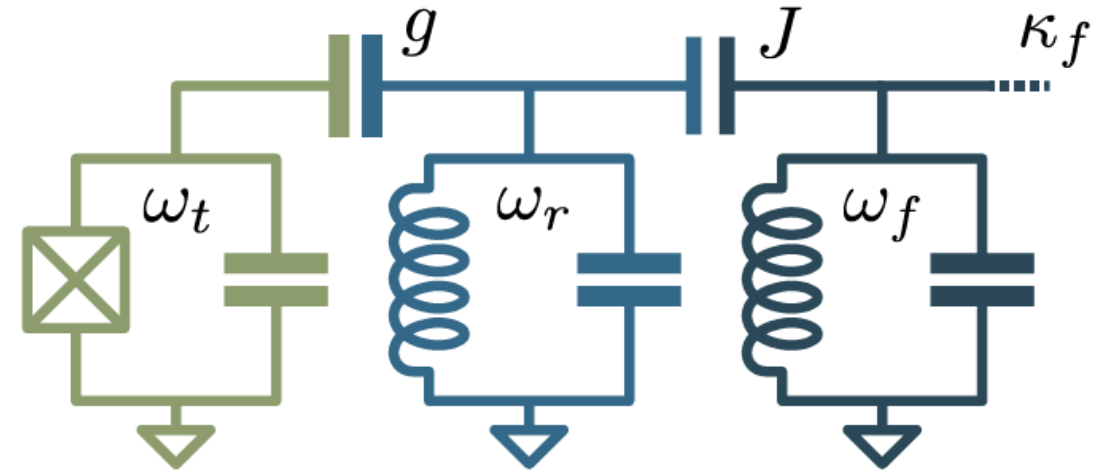


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- Optimisation with dynamiqs



System parameters

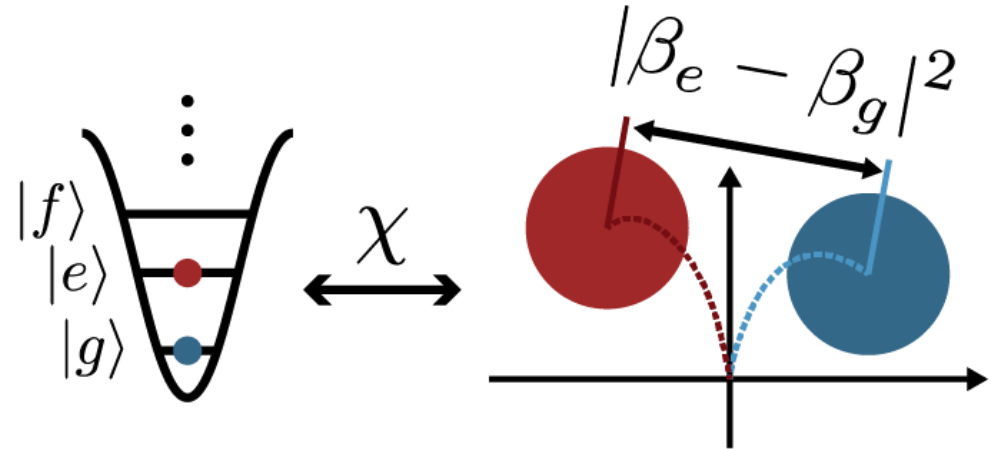
$E_J/2\pi = 16 \text{ GHz}$	$\omega_t/2\pi = 6 \text{ GHz}$	$\kappa_p/2\pi = 30 \text{ MHz}$	$g/2\pi = 150 \text{ MHz}$
$E_c/2\pi = 315 \text{ MHz}$	$\omega_r/2\pi = 7.2 \text{ GHz}$	$\kappa_q/2\pi = 8 \text{ KHz}$	$J/2\pi = 30 \text{ MHz}$
$E_J/E_c \approx 51$	$\omega_p/2\pi = 7.21 \text{ GHz}$	$\bar{n}_{\text{crit}} = 16$	

Optimizing transmon readout



Signal-to-noise ratio (Bultink et al., 2017)

$$\text{SNR} = \sqrt{2\eta\kappa_f \int_0^{\tau_m} dt |\beta_e - \beta_g|^2}$$





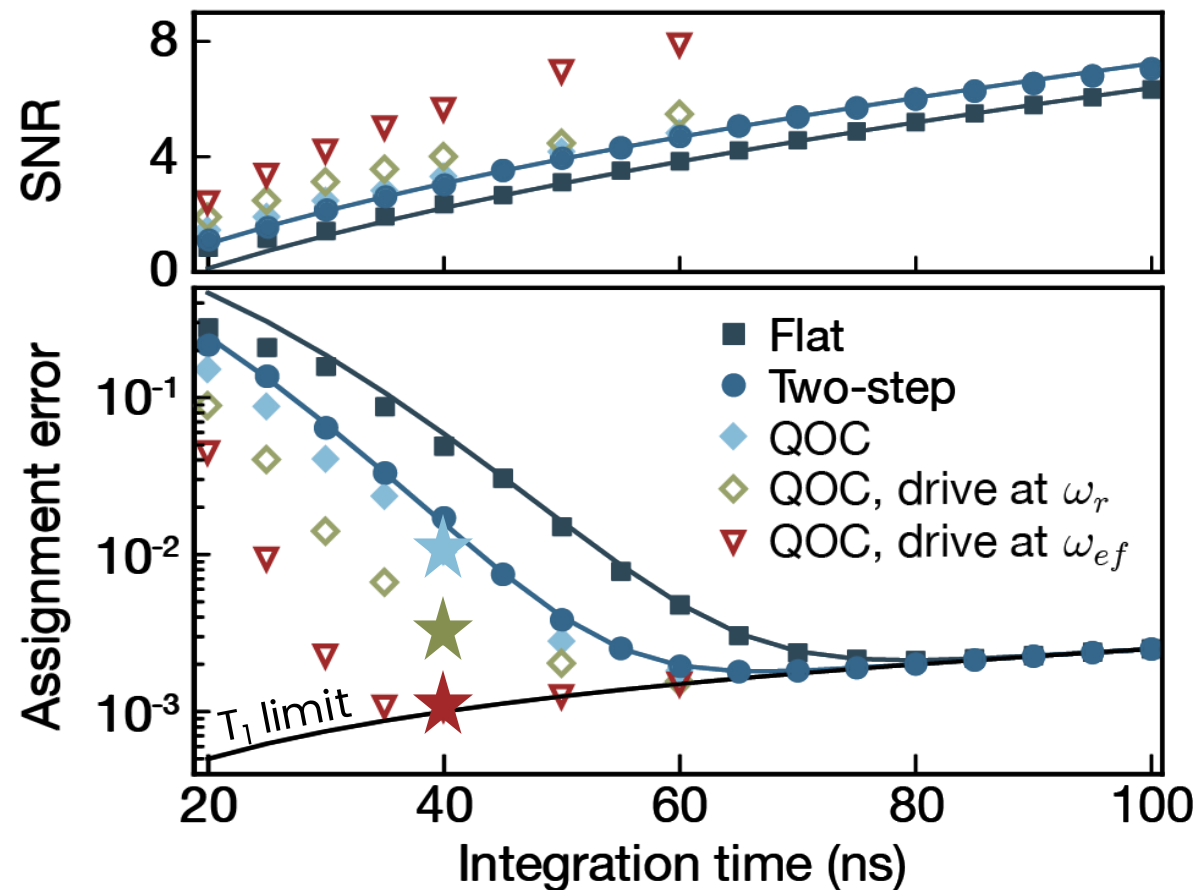
Optimizing transmon readout



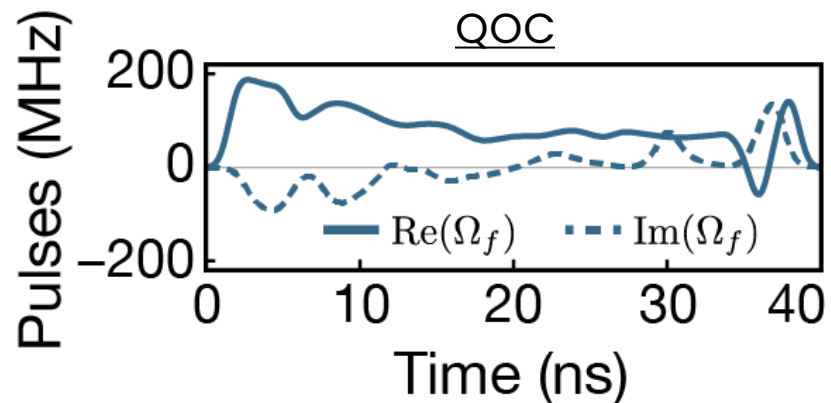
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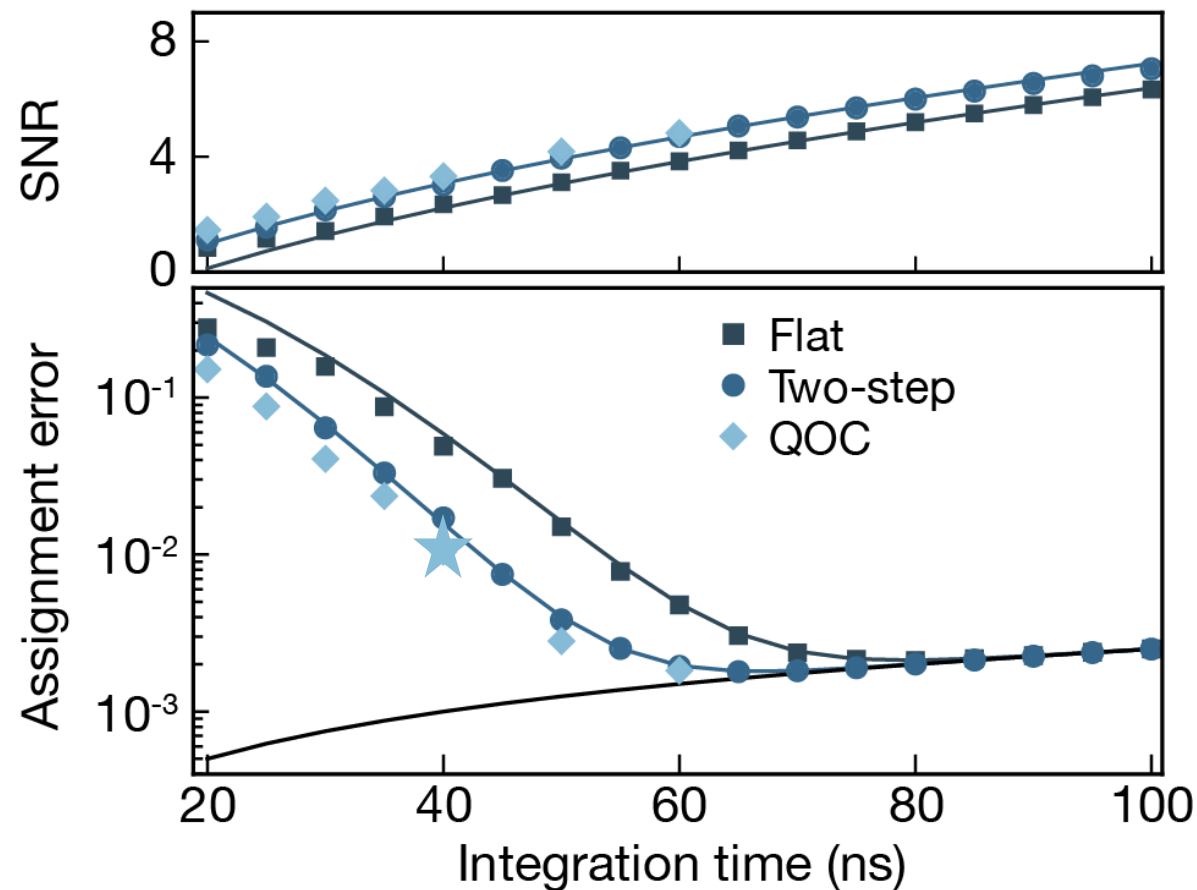
- 2 reference pulses
 - Flat pulse 
 - Two-step pulse 
- 3 optimized pulses
 - Drive filter
 - Drive filter + transmon @ ω_r
 - Drive filter + transmon @ ω_{ef}
- Optimize pulse envelopes + carrier frequencies
- Fair comparison: limit $n < n_{\text{crit}}$



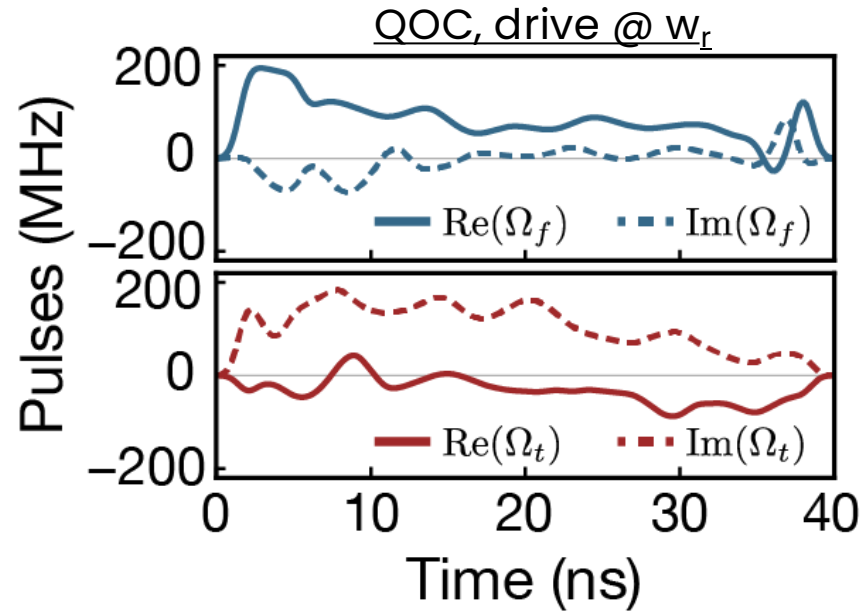
Optimizing towards a two-step pulse



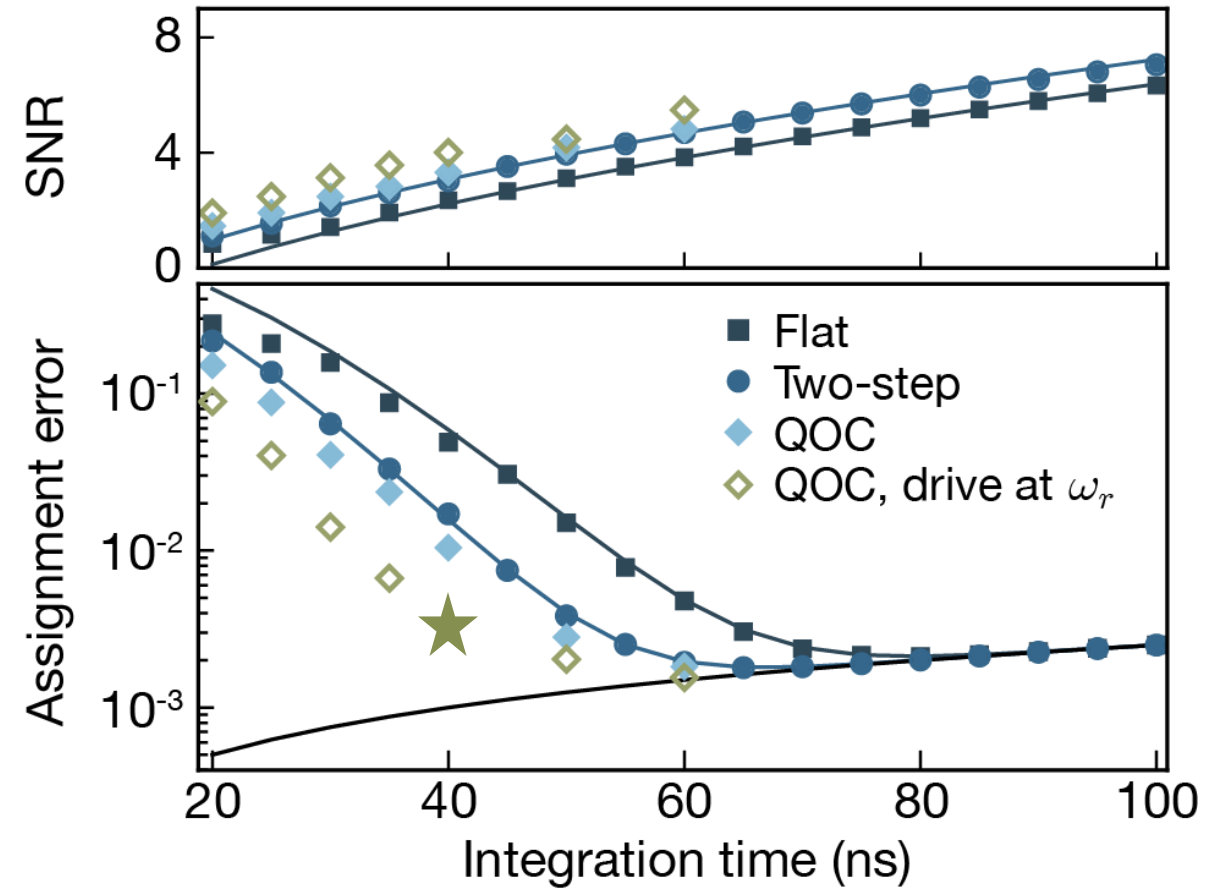
- Envelope similar to two-step (Walter, PRApplied 2017)
- Already optimal
- Limited by strength of dispersive coupling



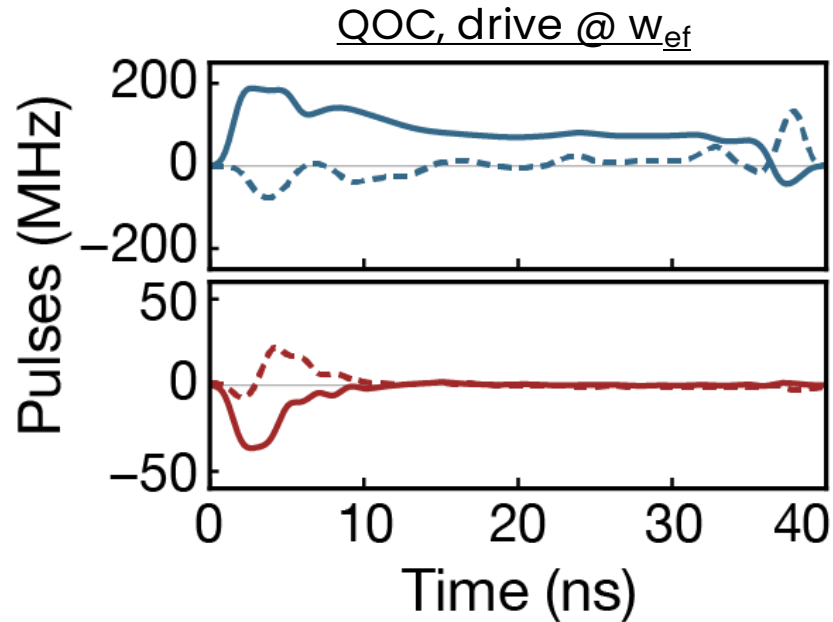
Multichannel driving



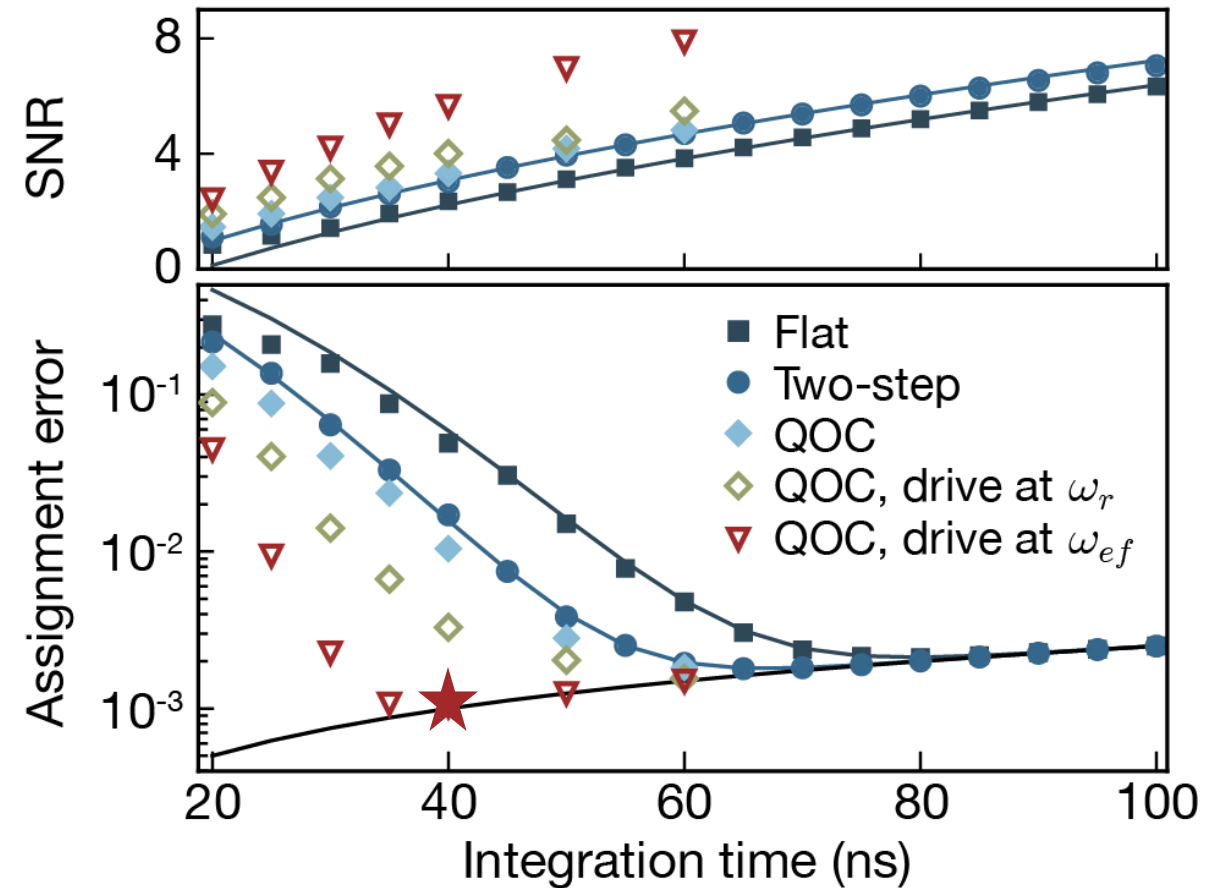
- Need another ingredient
- Similar to (Ikonen, PRL 2019) & (Touzard, PRL 2019)
- Drive transmon at $\omega_r \rightarrow$ displace origin of resonator phase-space



Shelving



- Shelving (Elder, PRX 2020) & (Hann, PRA 2018)
→ use $|f\rangle$ state with larger coupling
- Filter envelope similar to two-step
- 10ns pi-pulse with DRAG & stark-shift
- **x2 improvement in readout time**



01. Optimal control w/ **adjoint state method**:
low-memory, fast, generic

02. **Fast transmon readout** with
additional drive on the transmon

03. **Realistic pulses** and known
strategies found by optimizer

ALICE & BOB TALKS



Purcell filter trajectories



$|g\rangle$ and $|e\rangle$ trajectories in the Purcell filter \rightarrow enhanced integrated distance

