Combined Dissipative and Hamiltonian Confinement of Cat Qubits

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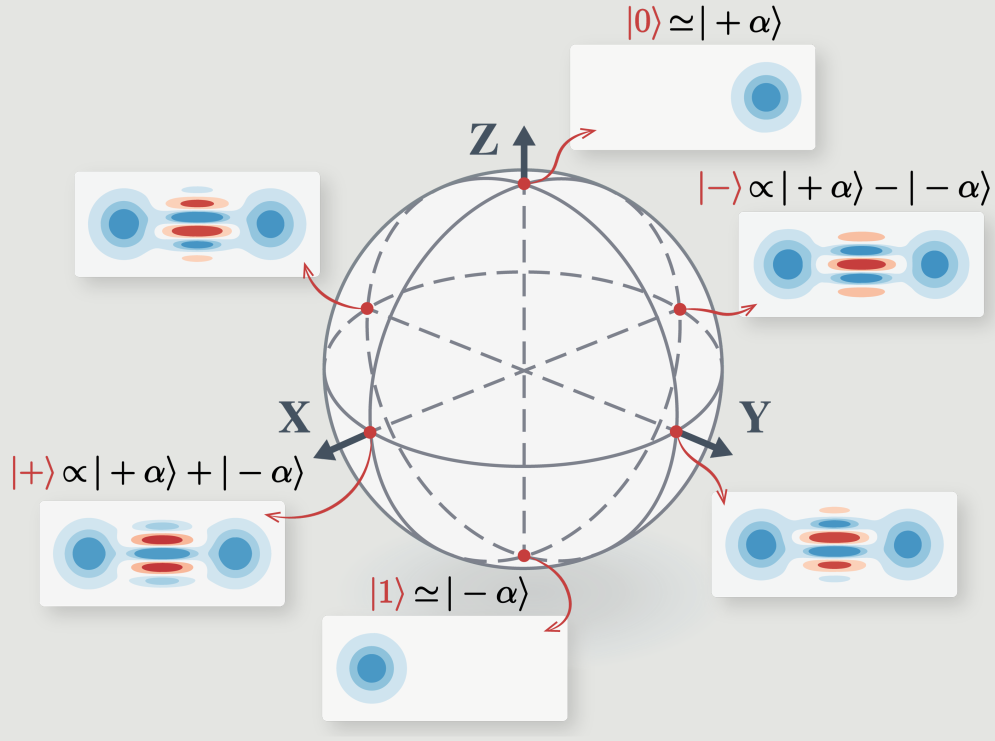
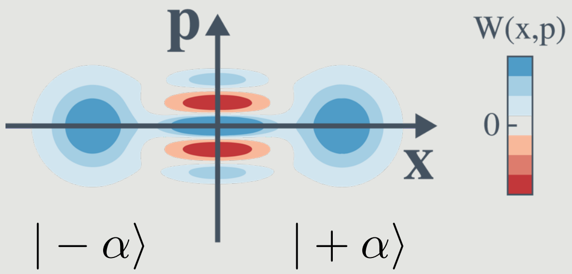
arXiv:2112.05545 (to appear in PRX Quantum)

Cat qubits are exponentially noise-biased qubits



Bloch Sphere Representation of a Cat Qubit

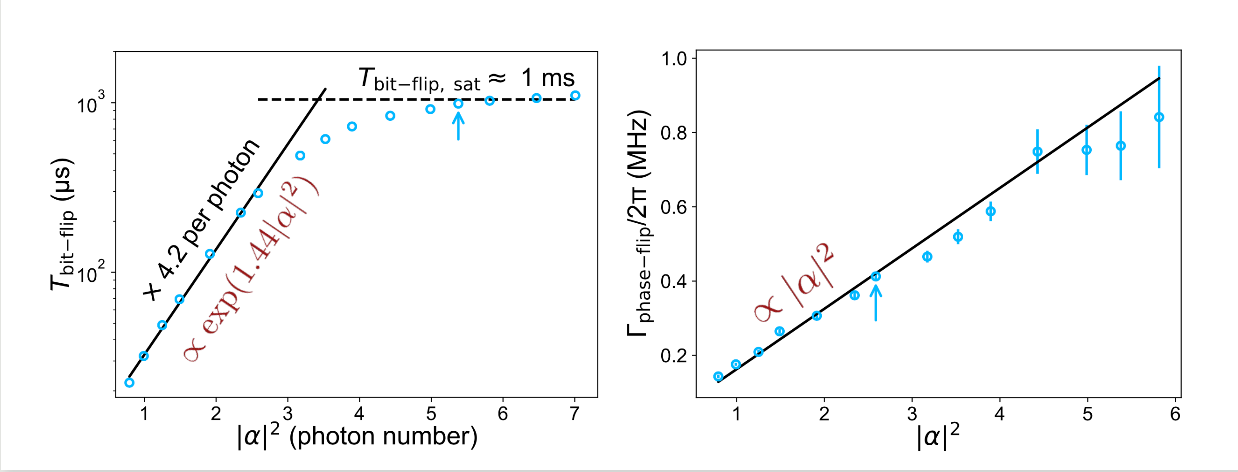
Cat states: coherent superposition of coherent states in a quantum oscillator



where $\hat{a}|\pm\alpha\rangle = \pm\alpha|\pm\alpha\rangle$

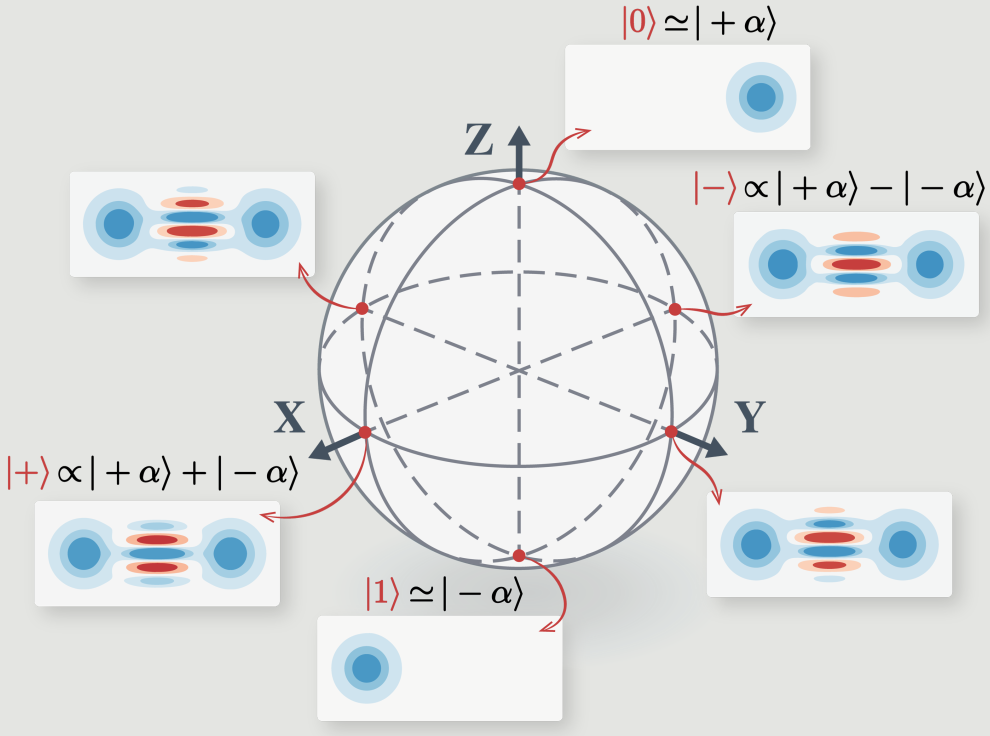
$\hat{a} = \hat{x} + i\hat{p}$

Cat qubits are exponentially noise-biased towards phase-flips



(Experimental data from Lescanne, Leghtas *et al., 2019*)

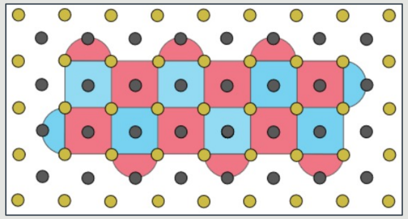
Fault-tolerent quantum computation with cat qubits

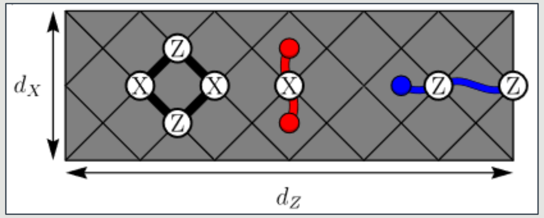


Bloch Sphere Representation of a Cat Qubit

For biased-noise qubits, quantum error correction (QEC) is simpler than for regular qubits

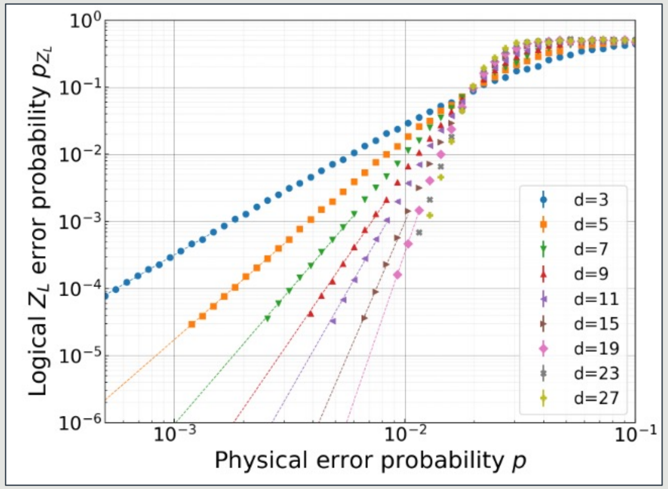
- Elongated surface code
- XZZX surface code
- 1D Repetition code





To reach QEC thresholds, there are mainly three pathways:

- Improve engineering
  $\kappa_1 \to 0$
- Improve QEC codes
  $p_{\mathrm{th}} \to 1$
- Improve gate designs
  $p_{\mathrm{gate}} \to 0$



Confining a cat qubit with engineered Hamiltonians or dissipation

To confine an oscillator to the cat qubit codespace, two main approaches exist. $\dot{\rho} = \hat{\mathcal{L}}\rho$

Two-photon dissipation $\hat{\mathcal{L}}_2 = \kappa_2 \mathcal{D}[\hat{a}^2 - \alpha^2]$

- C_α is a subspace of fixed points

$$\hat{\mathcal{L}}_2 \hat{\rho} = 0 \quad (\forall \hat{\rho} \in \mathrm{C}_\alpha)$$

- Any initial state converges asymptotically towards C_α

$$\hat{\rho}(t) \xrightarrow[t\to\infty]{} \hat{\rho}_\infty \in \mathrm{C}_\alpha$$

Autonomous stabilization

Kerr Hamiltonian $\hat{\mathcal{L}} = i\left[\hat{H}_{\mathrm{Kerr}}, \cdot\right]$

where $\hat{H}_{\mathrm{Kerr}} = K(\hat{a}^{\dagger 2} - \alpha^{*2})(\hat{a}^2 - \alpha^2)$

- $|\pm\alpha\rangle$ are degenerate eigenstates

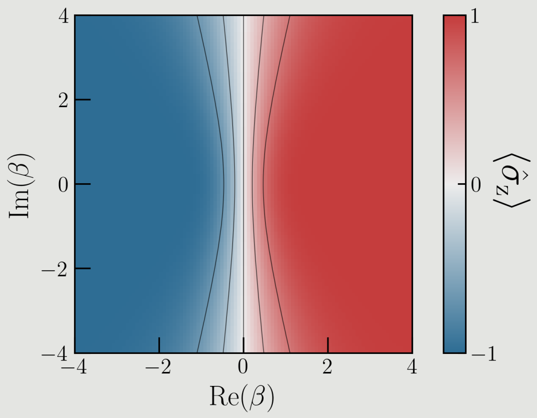
$$\hat{H}_{\mathrm{Kerr}} |\pm\alpha\rangle \propto |\pm\alpha\rangle$$

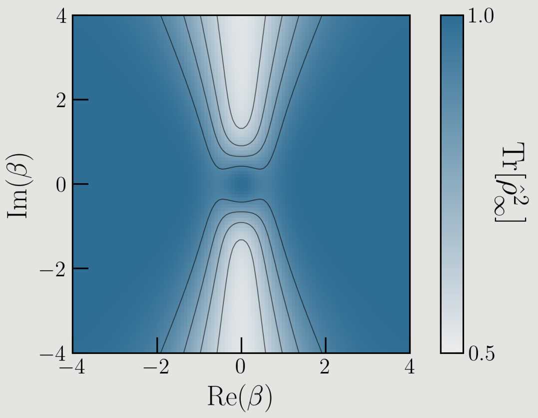
- $|\pm\alpha\rangle$ are gapped from other eigenstates

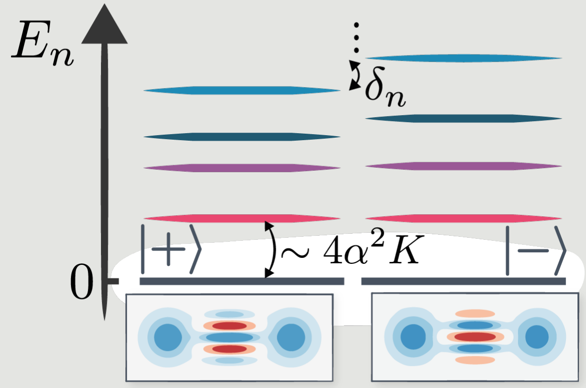
$$|E_{|\pm\alpha\rangle} - E_{|\psi\rangle}| \gg \kappa_{\mathrm{noise}}$$

Gap protection *(adiabatic theorem, perturbation theory)*

$\mathrm{C}_\alpha = \mathrm{span}\{|+\alpha\rangle\langle+\alpha|, |+\alpha\rangle\langle-\alpha|, |-\alpha\rangle\langle+\alpha|, |-\alpha\rangle\langle-\alpha|\}$







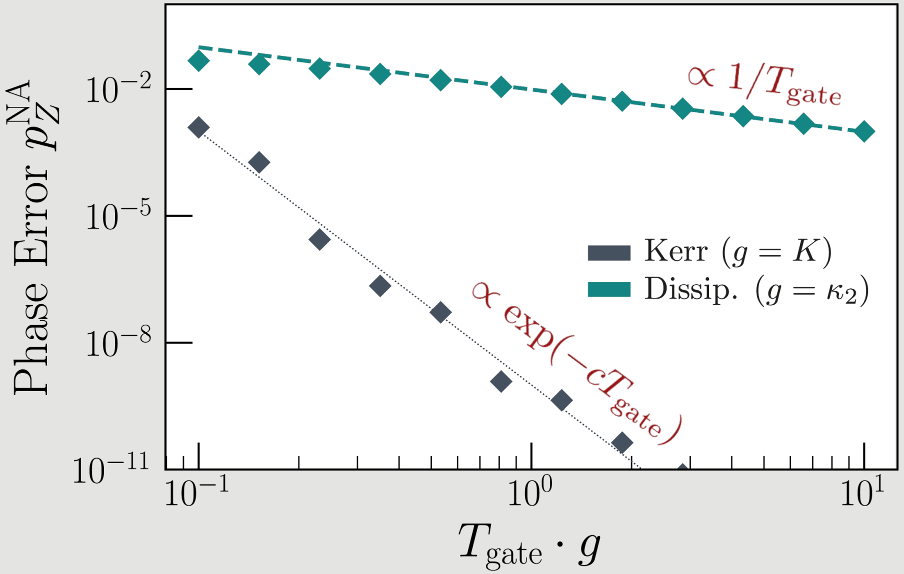
Mirrahimi *et al.*, NJP (2014); Puri *et al.*, npj QI (2017)

Pros and cons of Hamiltonian and dissipative confinement

Kerr confinement provides low-error gate designs, but is subject to thermal and dephasing noise.

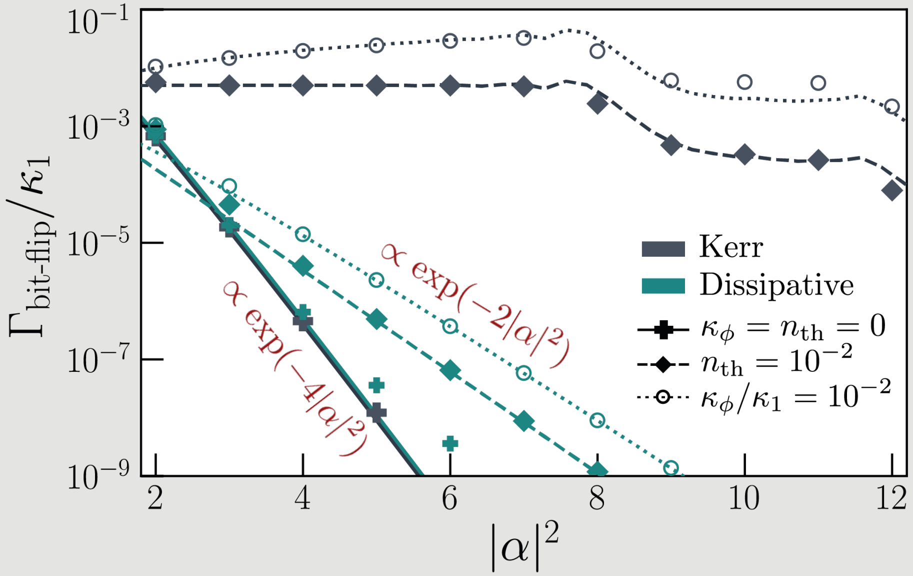
Single-qubit Z gate (noiseless)

$$\dot{\rho} = \mathcal{L}\rho - i\left[\varepsilon_Z(t)\hat{a}^\dagger + \varepsilon_Z^*(t)\hat{a}, \rho\right]$$



Idling qubit (noisy)

$$\dot{\rho} = \mathcal{L}\rho + \kappa_1(1 + n_{\mathrm{th}})\mathcal{D}[\hat{a}]\rho + \kappa_1 n_{\mathrm{th}}\mathcal{D}[\hat{a}^\dagger]\rho + \kappa_\phi\mathcal{D}[\hat{a}^\dagger\hat{a}]\rho$$



- Dissipative: linear scalir … l suppression at any $|\alpha|^2$
- Kerr: exponential scalin … ression up to $\mathcal{O}(\kappa_l)$
  up to $\mathcal{O}(\kappa_l)$

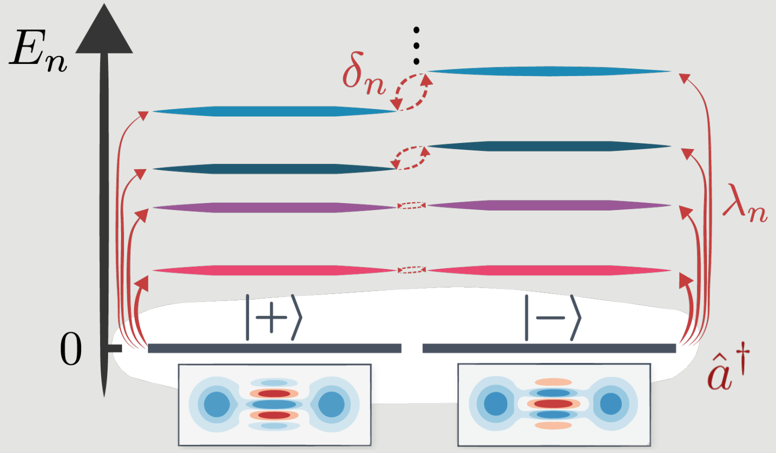
Why is the Kerr not exponentially protected?

How to benefit from the best of both?

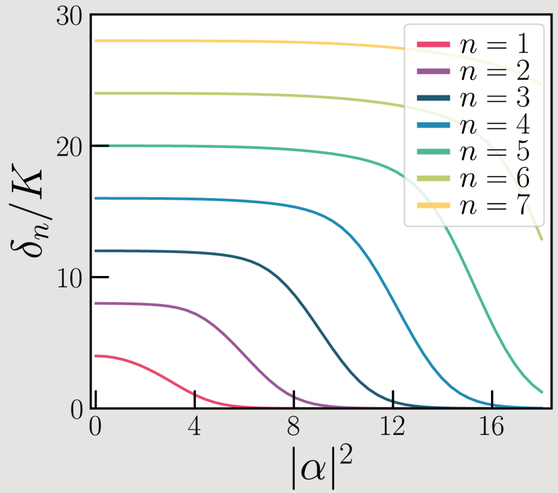
Putterman *et al.*, arXiv (2021); Xu *et al.*, arXiv (2021); Gautier *et al.*, arXiv (2021)

Why is the Kerr not exponentially protected?

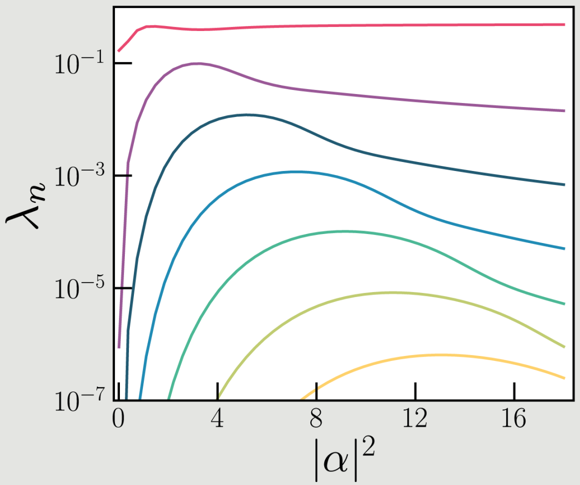
A realistic Kerr cat qubit is subject to $\dot{\rho} = \mathcal{L}_{\mathrm{Kerr}}\rho + \kappa_1(1+n_{\mathrm{th}})\mathcal{D}[\hat{a}]\rho + \kappa_1 n_{\mathrm{th}}\mathcal{D}[\hat{a}^\dagger]\rho + \kappa_\phi\mathcal{D}[\hat{a}^\dagger\hat{a}]\rho$



1. System initially in the cat codespace
2. At t=0, thermal excitation event with probability rate $\kappa_l = \kappa_1 n_{\mathrm{th}} + |\alpha|^2\kappa_\phi$
3. All Kerr eigenstates are populated
4. Dephasing of +/- branches induces bit-flip



- Suppressed exponentially for $|\alpha|^2 \gtrsim 4n$
- Diverge with n at $|\alpha|^2 < 4n$

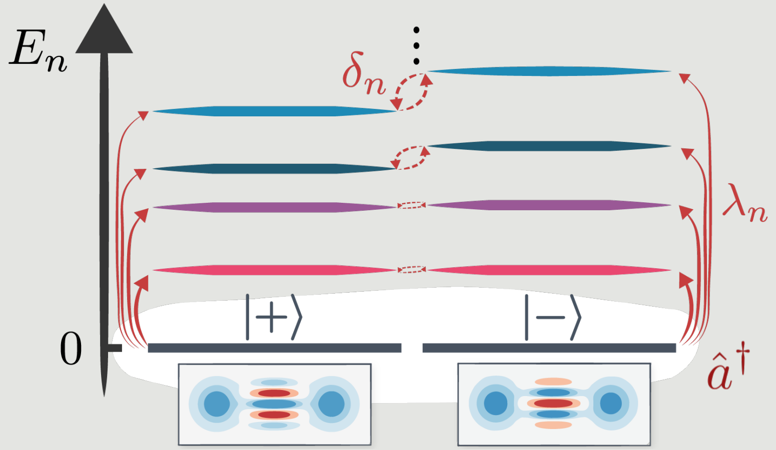


- $\lambda_n = \sum_{\pm} |\langle\phi_n^{\pm}|\hat{a}^\dagger|\alpha\rangle|^2/2$
- Not exponentially small !

Gautier *et al.*, arXiv (2021)

Why is the Kerr not exponentially protected?

A realistic Kerr cat qubit is subject to $\dot{\rho} = \mathcal{L}_{\mathrm{Kerr}}\rho + \kappa_1(1+n_{\mathrm{th}})\mathcal{D}[\hat{a}]\rho + \kappa_1 n_{\mathrm{th}}\mathcal{D}[\hat{a}^\dagger]\rho + \kappa_\phi \mathcal{D}[\hat{a}^\dagger\hat{a}]\rho$



1. System initially in the cat codespace
2. At t=0, thermal excitation event with probability rate $\kappa_l = \kappa_1 n_{\mathrm{th}} + |\alpha|^2\kappa_\phi$
3. All Kerr eigenstates are populated
4. Dephasing of +/- branches induces bit-flip

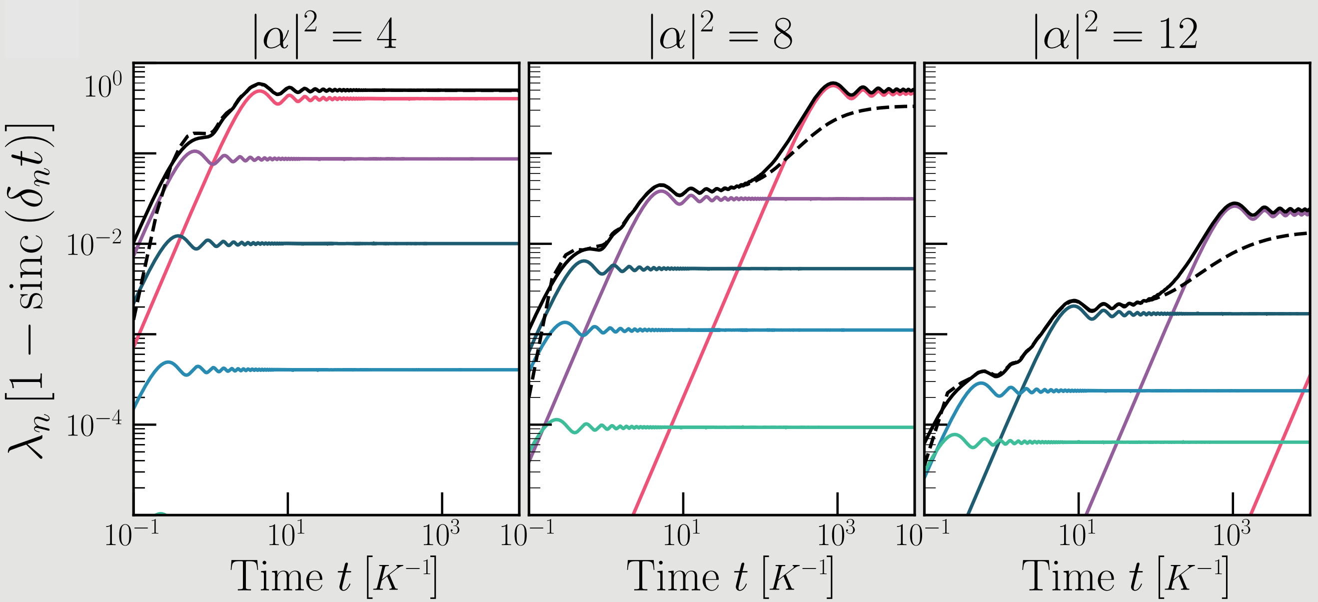
First-order estimation of bit-flip errors

leakage overlap dephasing

$$p_X(t) = \kappa_l t \sum_{n>0} \lambda_n [1 - \mathrm{sinc}(\delta_n t)]$$

$|\alpha|^2 = 12$
$t = 1/K$

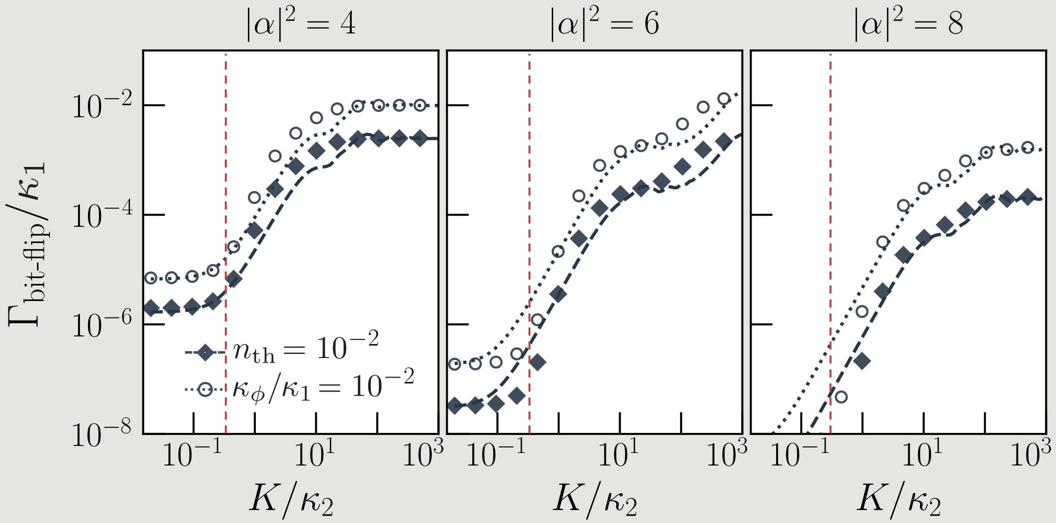
$p_X(t)/\kappa_l t \approx 4 \times 10^{-4}$
$\exp(-2|\alpha|^2) \approx 4 \times 10^{-11}$



How to benefit from the best of both worlds?

To benefit from the best of both worlds, why not simply combine both confinement methods?

$$\dot{\rho} = \hat{\mathcal{L}}_2\rho + \hat{\mathcal{L}}_{\mathrm{Kerr}}\rho + \cdots$$



Idling qubit

To conserve the exponential error bias

- $K/\kappa_2 \lesssim 0.3$

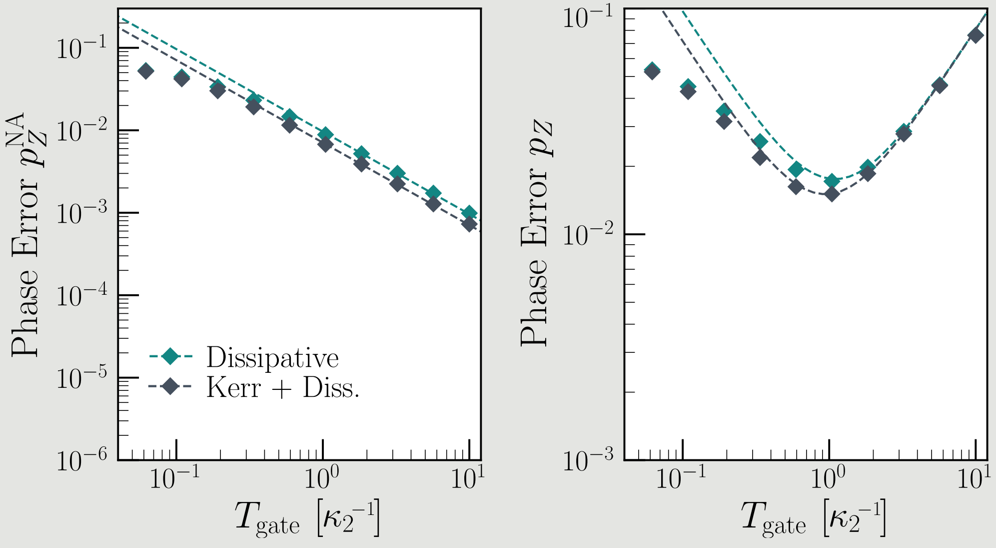
Single-qubit Z gate

Phase errors induced by the gate

Kerr prefactor | dissipative errors | cavity dephasing

- $$p_Z = \frac{1}{1+\frac{4K^2}{\kappa_2^2}} \frac{\pi^2}{16|\alpha|^4\kappa_2 T_{\mathrm{gate}}} + \kappa_1|\alpha|^2 T_{\mathrm{gate}}$$

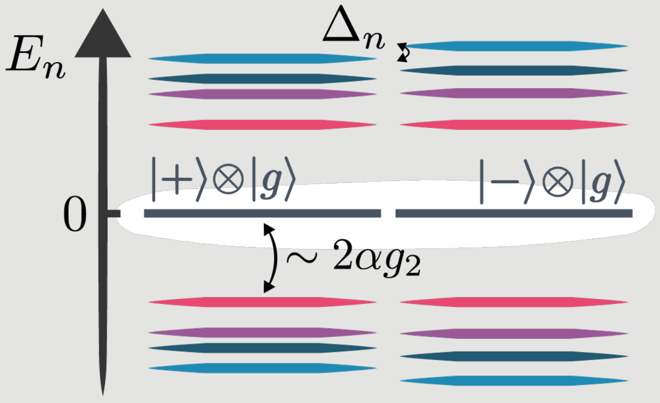
- Minimal gain



Confinement with a Two-Photon Exchange Hamiltonian

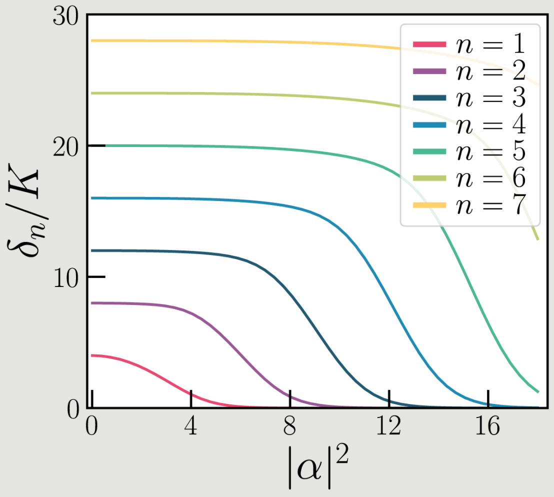
New cat qubit Hamiltonian confinement coined Two-Photon Exchange (TPE)

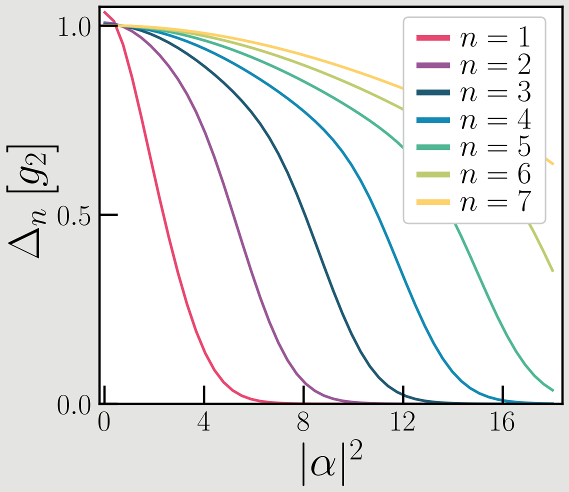
$$\hat{H}_{\mathrm{TPE}} = g_2(\hat{a}^2 - \alpha^2)\hat{\sigma}_+ + \mathrm{h.c.}$$



- Gapped Hamiltonian
- Degenerate subspace given by the cat qubit
- $(\hat{H}_{\mathrm{TPE}}/g_2)^2 = (\hat{a}^2 - \alpha^2)^\dagger(\hat{a}^2 - \alpha^2)|g\rangle\langle g| + (\hat{a}^2 - \alpha^2)(\hat{a}^2 - \alpha^2)^\dagger|e\rangle\langle e|$
  $= (\hat{H}_{\mathrm{Kerr}}/K)|g\rangle\langle g| + (\hat{H}'_{\mathrm{Kerr}}/K)|e\rangle\langle e|$

$$\text{“TPE”} = \sqrt{\text{“Kerr”}}$$





- Suppressed exponentially for $|\alpha|^2 \gtrsim 4n$
- Bounded by g_2 at $|\alpha|^2 < 4n$

Gautier *et al.*, arXiv (2021)

How to benefit from the best of both worlds?

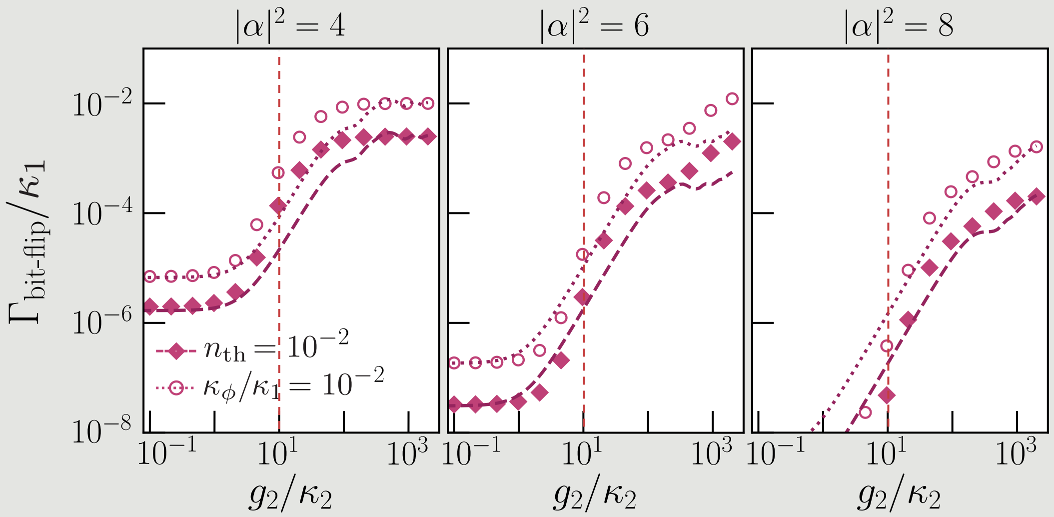
To benefit from the best of both worlds, it is better to combine TPE and dissipation confinement !

$$\dot{\rho} = \hat{\mathcal{L}}_2\rho + \hat{\mathcal{L}}_{\mathrm{TPE}}\rho + \cdots$$

Idling qubit

To conserve the exponential error bias

- $g_2/\kappa_2 \lesssim 10$



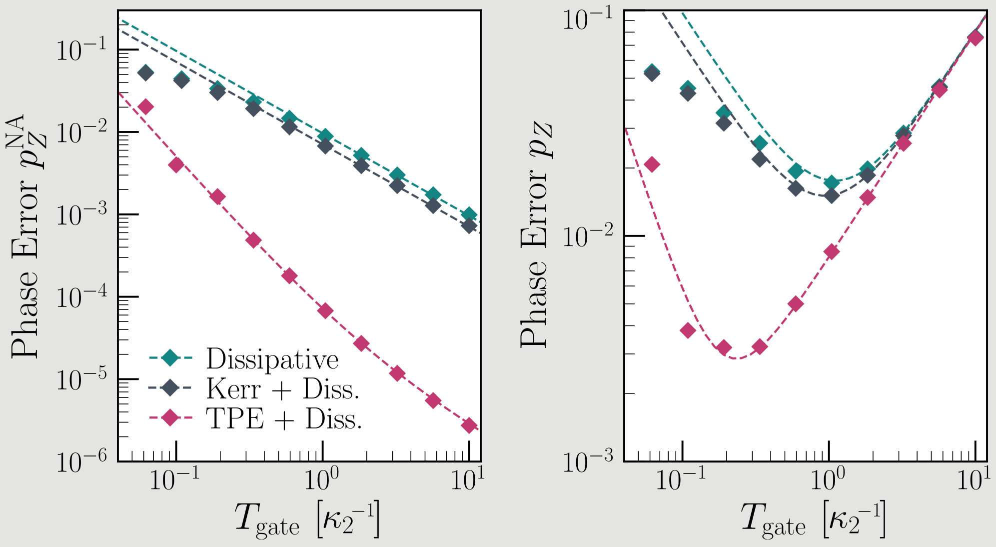
Single-qubit Z gate

Phase errors induced by the gate

TPE prefactor | dissipative errors | cavity dephasing

$$p_Z = \frac{1}{1+\frac{4g_2^2}{\kappa_2^2}} \frac{\pi^2}{16|\alpha|^4\kappa_2 T_{\text{gate}}} + \kappa_1|\alpha|^2 T_{\text{gate}}$$

- x400 gain on gate-induced errors



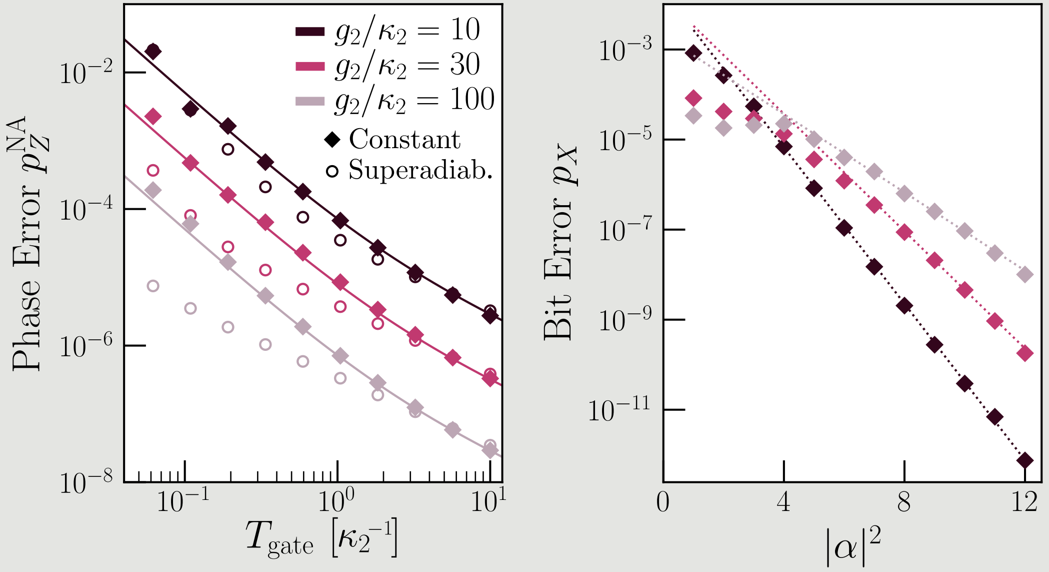
Gautier *et al.*, arXiv (2021)

Combined TPE confinement for gate engineering

Away from the bias-preserving point, gate-induced errors can further be reduced at the cost of additional bit-flip errors.

Single-qubit Z gate

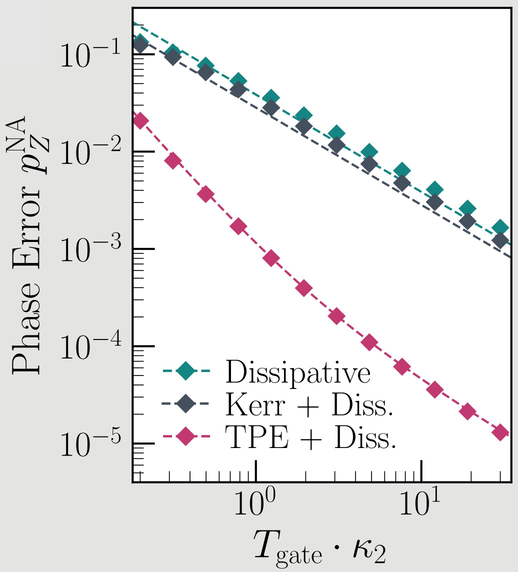
$$\dot{\rho} = \mathcal{L}\rho - i\left[\varepsilon_Z(t)\hat{a}^\dagger + \varepsilon_Z^*(t)\hat{a}, \rho\right]$$



$$p_Z = \frac{1}{1 + \frac{4g_2^2}{\kappa_2^2}} \frac{\pi^2}{16|\alpha|^4\kappa_2 T_{\mathrm{gate}}}$$

Two-qubit CNOT gate

$$\dot{\rho} = g\mathcal{L}_{\mathrm{conf}}^{(co)}\rho - i[\hat{H}_{CX}, \rho]$$



- Up to x100 two-qubit gate fidelity improvement
- Reduced leakage compared to dissipative gate designs
- Repetition code threshold: 0.7% (Diss.) ➡ 2% (TPE + Diss.)

Gautier *et al.*, arXiv (2021). Threshold calculation by Jérémie Guillaud.

Engineering a combined TPE and dissipative confinement

Potential energy of the ATS
(Assymetrically Threaded SQUID)

$$U(\varphi) = \frac{1}{2}E_L\varphi^2 - 2E_J\left[\varepsilon(t)\sin(\varphi) - \eta\cos(\varphi)\right]$$

$$\hat{H} = g_2(\hat{a}^2 - \alpha^2)\hat{b}^\dagger + \text{h.c.}$$

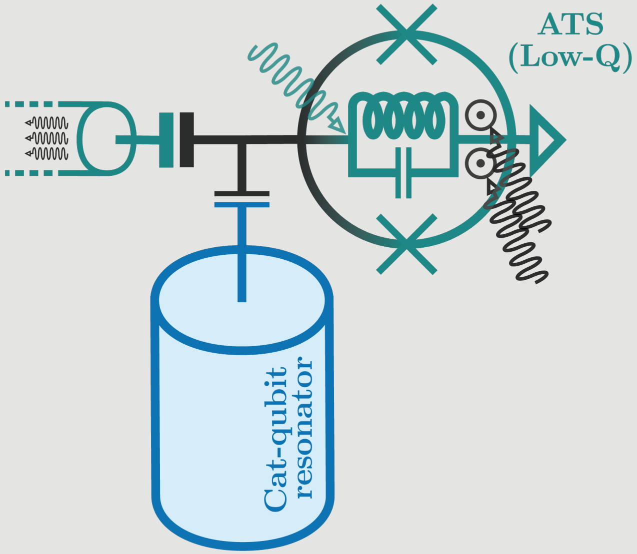
$\kappa_b \gg g_2$

$$\kappa_2\mathcal{D}[\hat{a}^2 - \alpha^2]$$

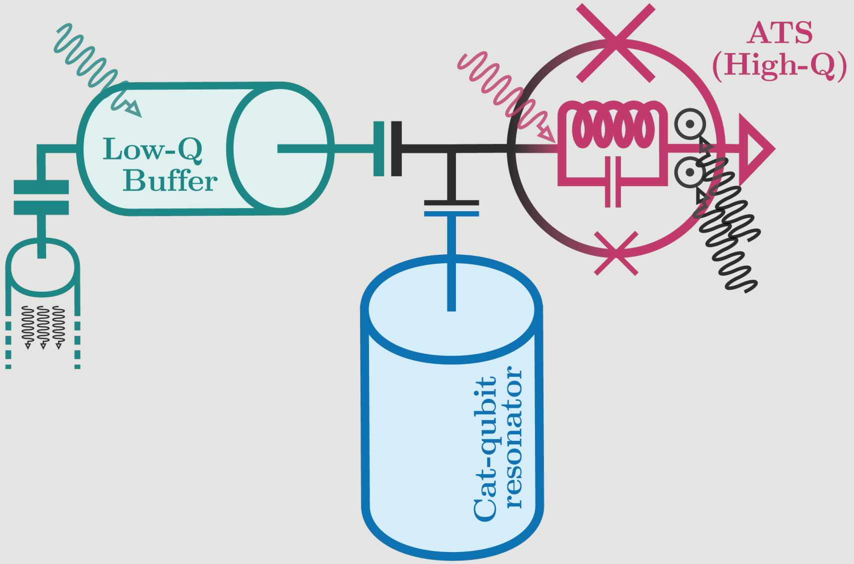
$$\hat{H} = g_{2,l}(\hat{a}^2 - \alpha^2)\hat{b}_\text{l}^\dagger + \text{h.c.} + g_{2,h}(\hat{a}^2 - \alpha^2)\hat{b}_\text{h}^\dagger + \text{h.c.} - \chi_{hh}\hat{b}_\text{h}^{\dagger 2}\hat{b}_\text{h}^2$$

$\kappa_b \gg g_{2,l}$: $\kappa_2\mathcal{D}[\hat{a}^2 - \alpha^2]$

$\chi_{hh} \gg g_{2,h}$: $g_{2,h}(\hat{a}^2 - \alpha^2)\hat{\sigma}_+ + \text{h.c.}$



Dissipative cat qubit circuit design

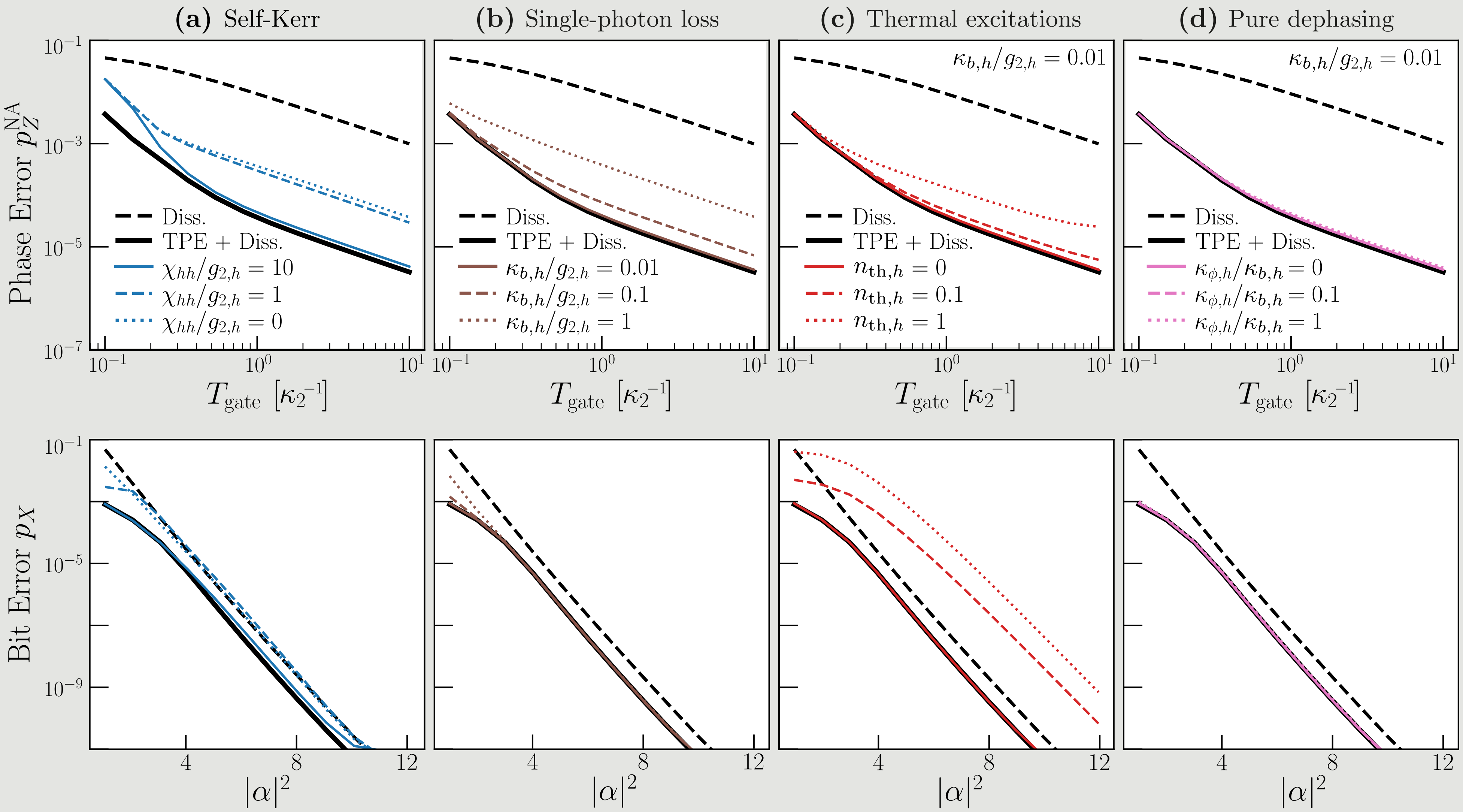


Combined TPE + Diss. circuit proposition

Lescanne *et al.*, Nat.Phy. (2019); Gautier *et al.*, arXiv (2021).

Thanks for your attention!

Effect of spurious buffer terms



Gautier *et al.*, arXiv (2021).