Designing High-Fidelity Gates for Dissipative Cat Qubits

Ronan Gautier^{1,2}, Mazyar Mirrahimi¹, Alain Sarlette^{1,3}

¹Laboratoire de Physique de l'École Normale Supérieure, Inria, ENS, Mines ParisTech, Sorbonne Université, Paris, France ²Institut quantique et Département de physique, Université de Sherbrooke, Sherbrooke, Canada ³Department of Electronics and Information Systems, Ghent University, Belgium

arXiv:2303.00760



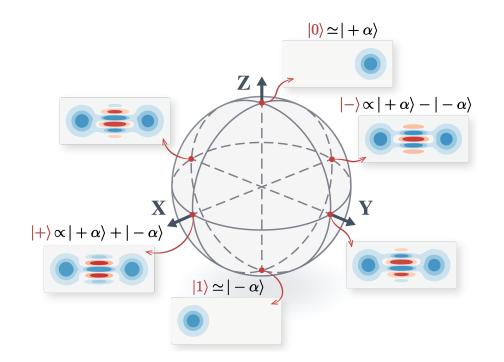
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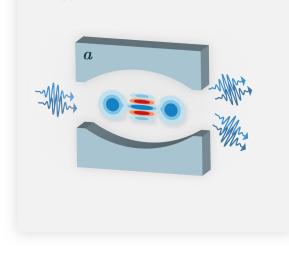
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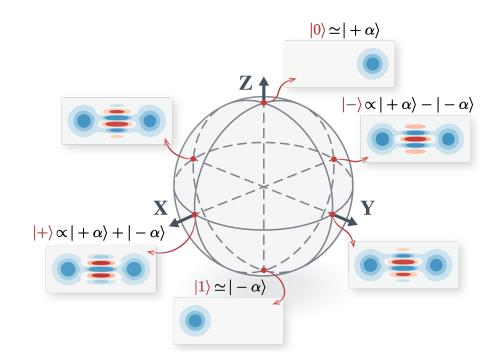


Cat qubits can be stabilized with two-photon dissipation, often mediated by an **ancillary buffer mode**

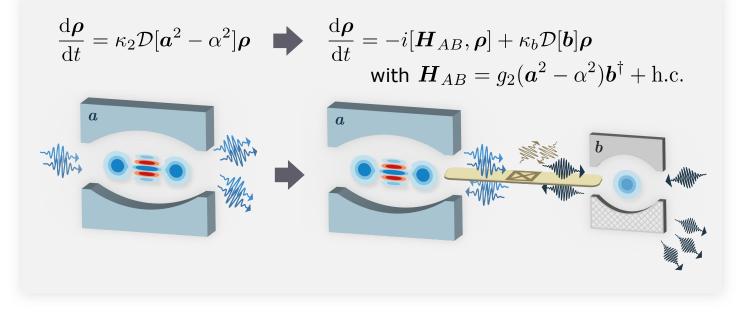
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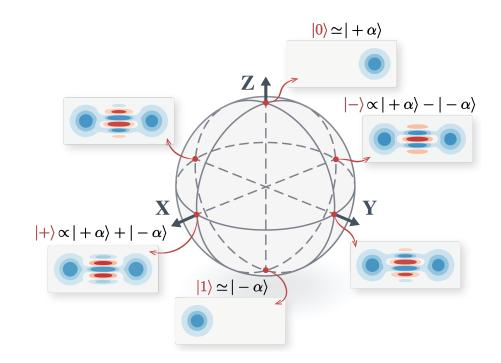
Bloch Sphere Representation of a Cat Qubit



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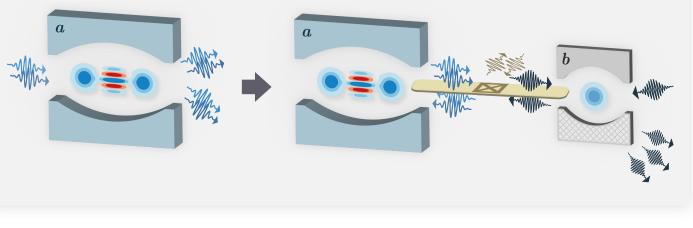
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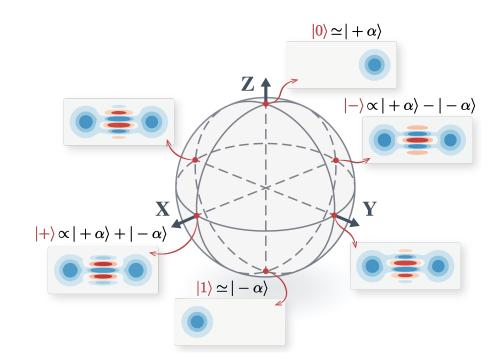
$$\frac{\mathrm{d}\boldsymbol{\rho}}{\mathrm{d}t} = \kappa_2 \mathcal{D}[\boldsymbol{a}^2 - \alpha^2]\boldsymbol{\rho} \quad \Longrightarrow \quad \frac{\mathrm{d}\boldsymbol{\rho}}{\mathrm{d}t} = -i[\boldsymbol{H}_{AB}, \boldsymbol{\rho}] + \kappa_b \mathcal{D}[\boldsymbol{b}]\boldsymbol{\rho}$$

with $\boldsymbol{H}_{AB} = g_2(\boldsymbol{a}^2 - \alpha^2)\boldsymbol{b}^{\dagger} + \mathrm{h.c.}$



• Adiabatic elimination
$$\kappa_2\equiv rac{4g_2^2}{\kappa_b}$$

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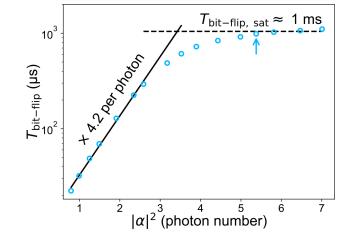
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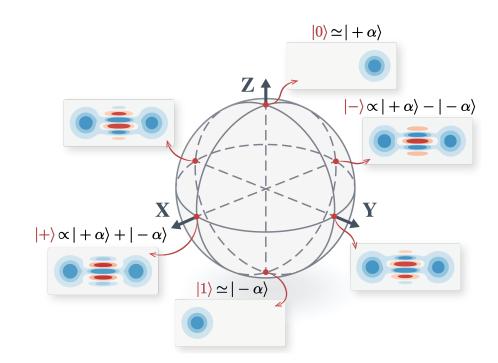
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- Adiabatic elimination $\kappa_2 \equiv rac{4g_2^2}{\kappa_b}$
- Exponentially noise-biased towards phase-flips



Cochrane et al., PRA (1999); Leghtas et al., PRA (2013); Mirrahimi et al., NJP (2014); Lescanne, Leghtas et al., Nat.Phy. (2019)



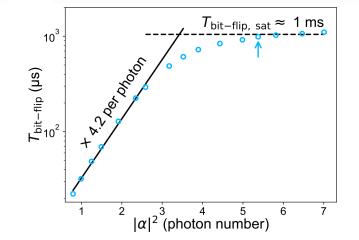
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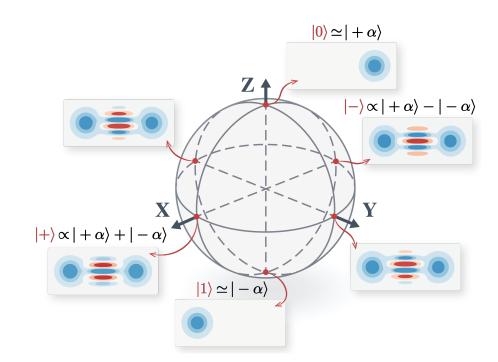
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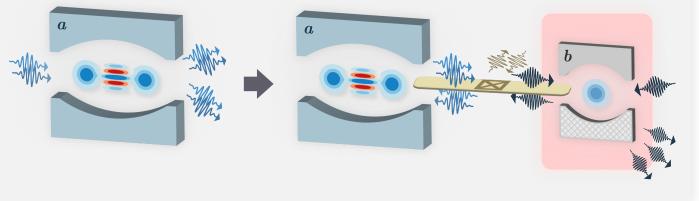
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$$rac{\partial t}{\partial t} = -\imath_[oldsymbol{H}_{AB},oldsymbol{
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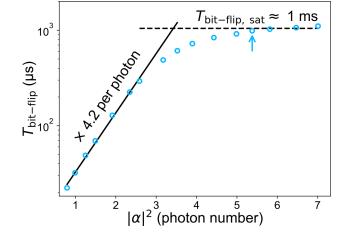
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 \mathbf{a} + $\mathbf{a} \mathcal{D}[\mathbf{b}] \mathbf{a}$

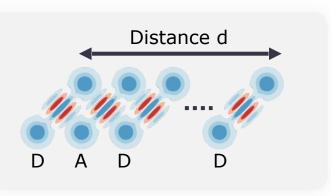


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➡ Gate engineering



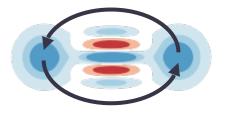
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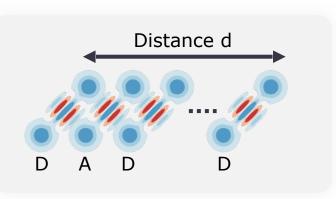
For fault-tolerant universal QC with repetition cat qubits, only four gates are required on top of preparation and measurement in $|\pm_L\rangle$

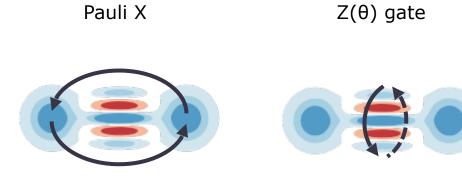
Distance d D A D D

Pauli X

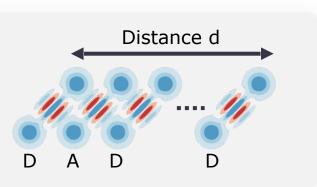


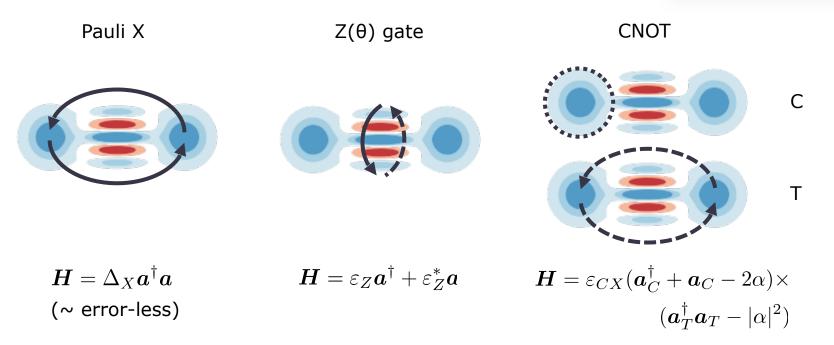
 $oldsymbol{H} = \Delta_X oldsymbol{a}^\dagger oldsymbol{a}$ (~ error-less)

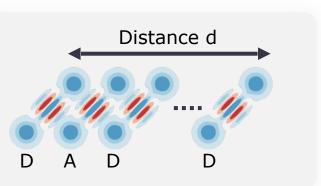


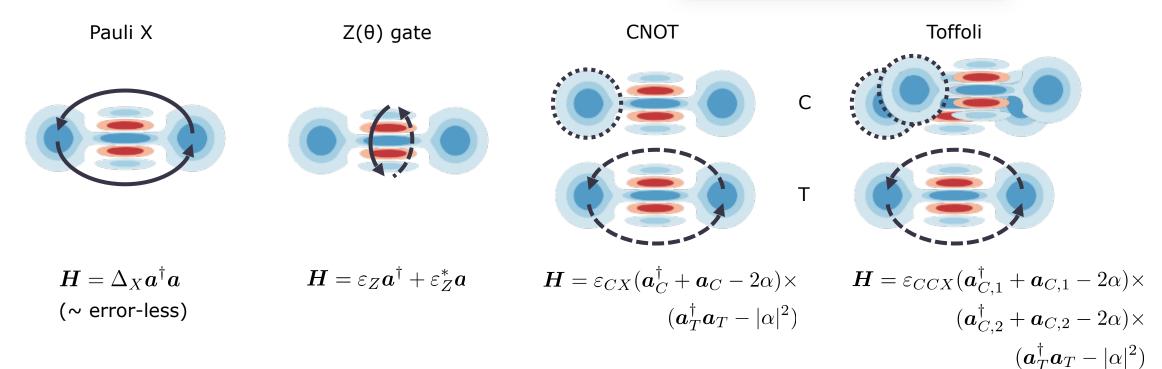


$$oldsymbol{H} = \Delta_X oldsymbol{a}^\dagger oldsymbol{a}$$
 $oldsymbol{H} = arepsilon_Z oldsymbol{a}^\dagger + arepsilon_Z^* oldsymbol{a}$ (~ error-less)

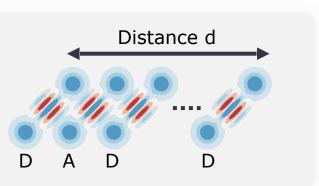


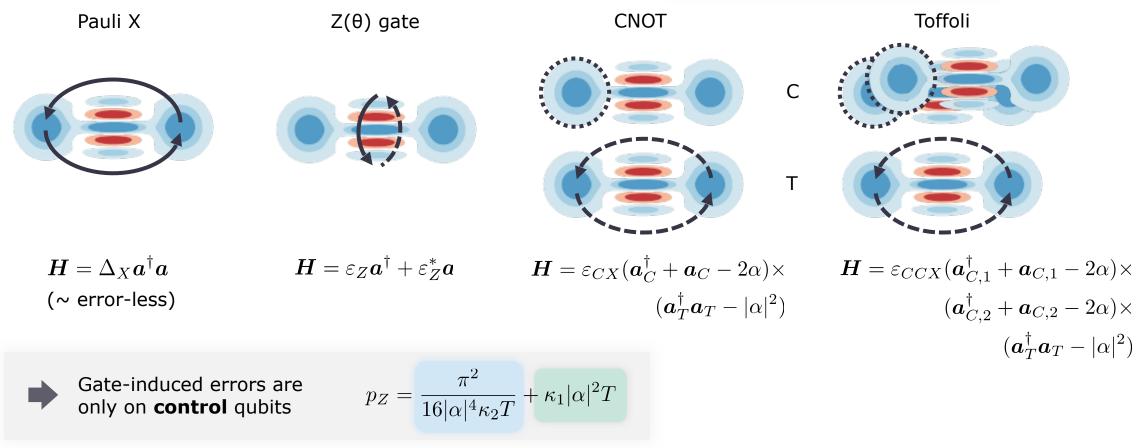






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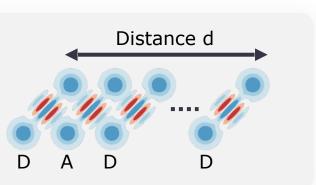


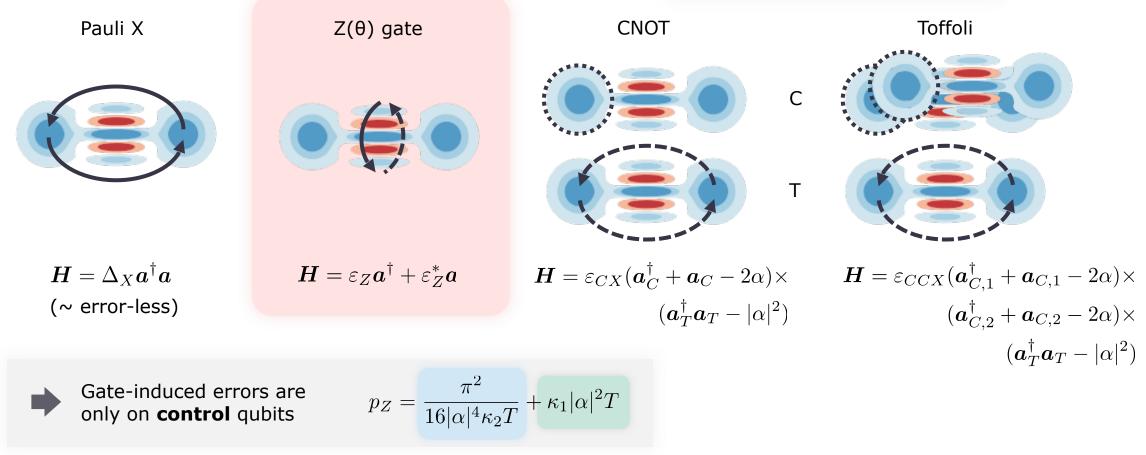


Gate errors Cavity lifetime

Guillaud et al., PRX (2019); Chamberland et al., PRX Q (2021)

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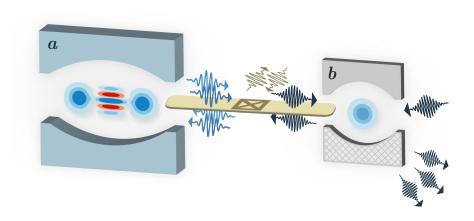


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Consider the Z gate Hamiltonian with a buffer mode,

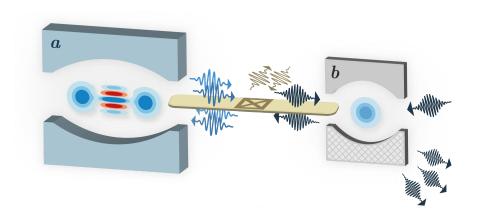
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Introduce the Shifted Fock Basis



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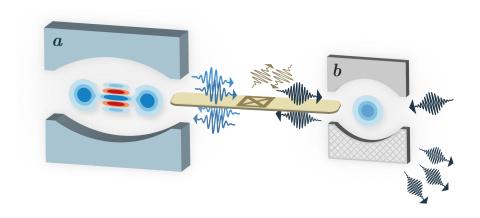
$$a \rightarrow \sigma_z \otimes (\tilde{a} + \alpha)$$

qubit gauge \bullet \bullet \bullet \bullet \bullet \bullet \bullet
 $H = g_2(\tilde{a}^2 + 2\alpha \tilde{a})b^{\dagger} + \varepsilon_Z \sigma_z(\tilde{a} + \alpha) + \text{h.c.}$

Write master equation in Heisenberg picture

$$\dot{\tilde{a}} = -2i\alpha g_2 b - i\varepsilon_Z \sigma_z$$

 $\dot{b} = -2i\alpha g_2 \tilde{a} - \kappa_b b/2$



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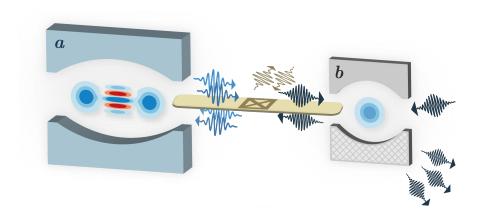
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Decouple these equations

$$\ddot{b} + \kappa \dot{b} + \omega_0^2 b = -\omega_0 \varepsilon_Z \sigma_z$$

with $\kappa = \kappa_b/2$ and $\omega_0 = 2\alpha g_2$



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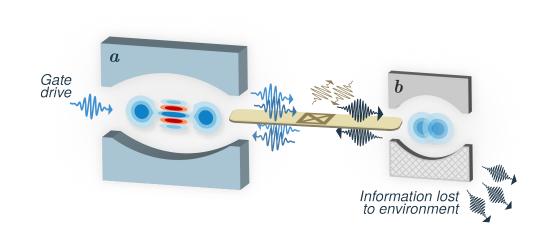
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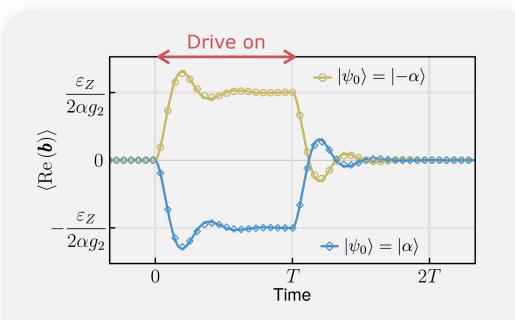
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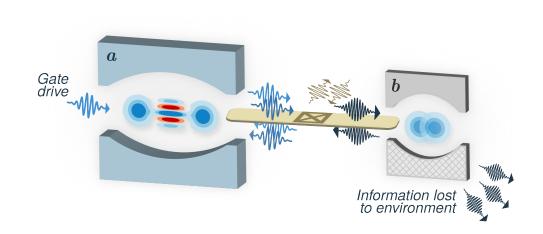
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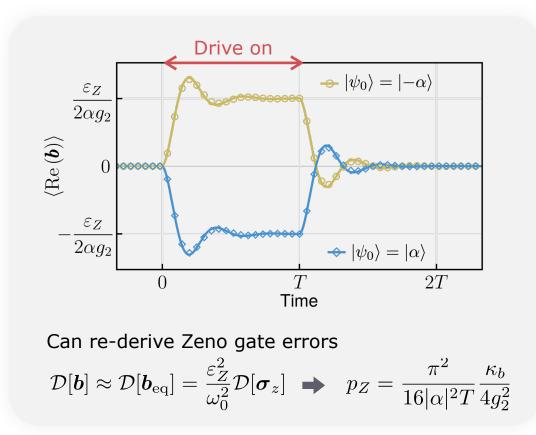
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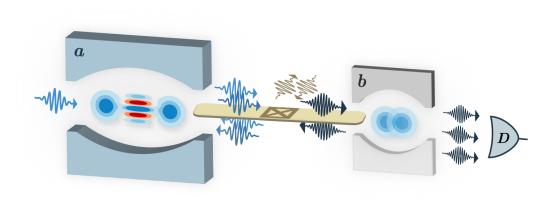
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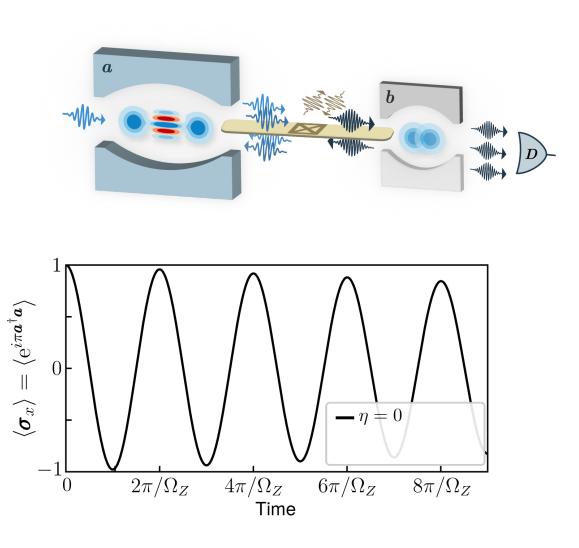
Information is lost through the buffer mode **measure the buffer output** to retrieve it

 $d\boldsymbol{\rho} = -i[\boldsymbol{H}, \boldsymbol{\rho}]dt + \kappa_b \left(\mathcal{D}_{\eta}[\boldsymbol{b}]\boldsymbol{\rho} dt + \mathcal{J}[\boldsymbol{b}]\boldsymbol{\rho} dN_{\eta} \right)$ no-jump jump with $0 \le \eta \le 1$ (detection efficiency)



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 \boldsymbol{a} No parity damping $\langle \boldsymbol{\sigma}_x
angle = \langle \mathrm{e}^{i \pi \boldsymbol{a}^\dagger \boldsymbol{a}}
angle$ 0 $\begin{array}{l} - \eta = 0 \\ - \eta = 1 \text{ (no jump)} \end{array}$ _1 $6\pi/\Omega_Z$ $2\pi/\Omega_Z$ $4\pi/\Omega_Z$ $8\pi/\Omega_Z$ 0 Time

Assuming $\eta=1$

• Preserves purity (no information lost)

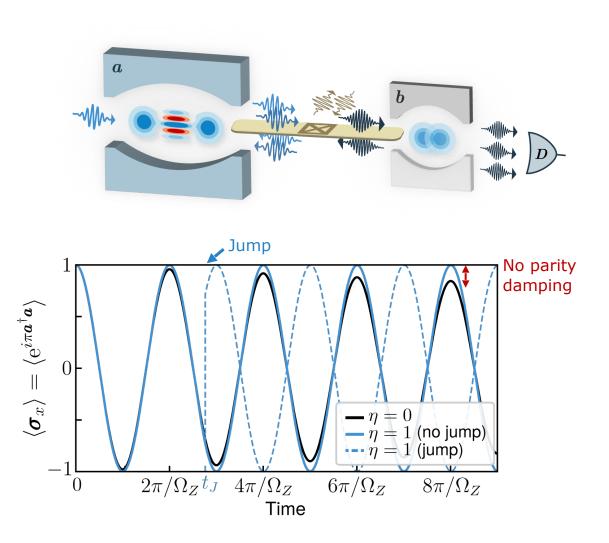
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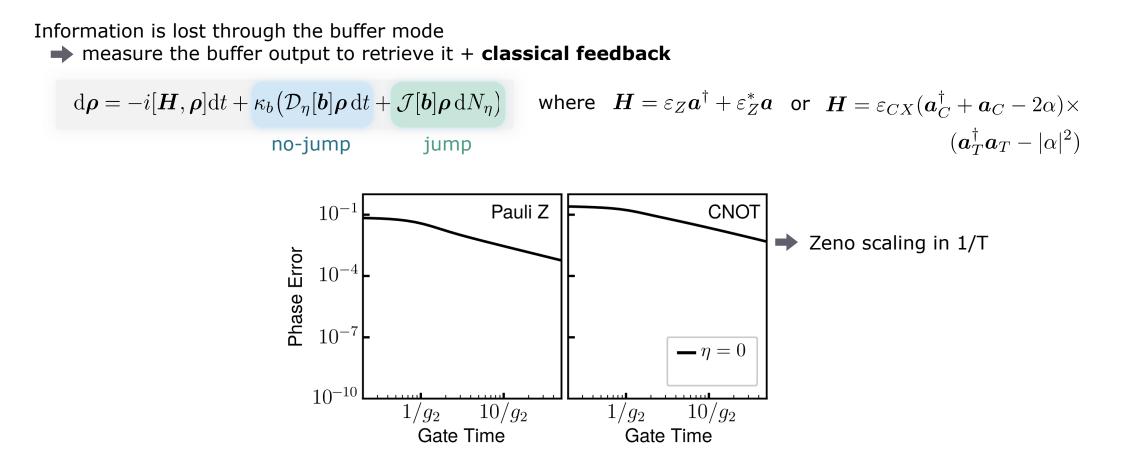
- Preserves purity (no information lost)
- Jump detected = parity swap

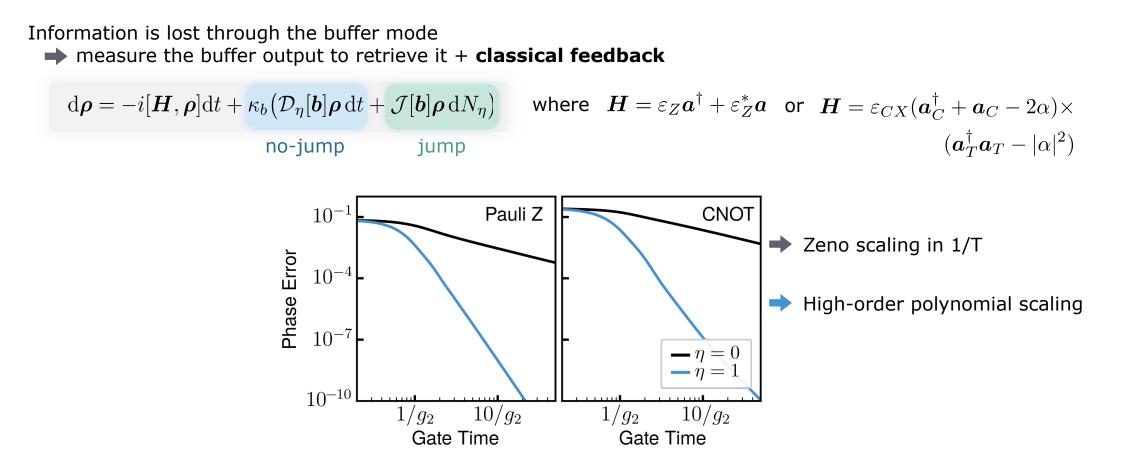
 $\bullet b \propto \sigma_z$

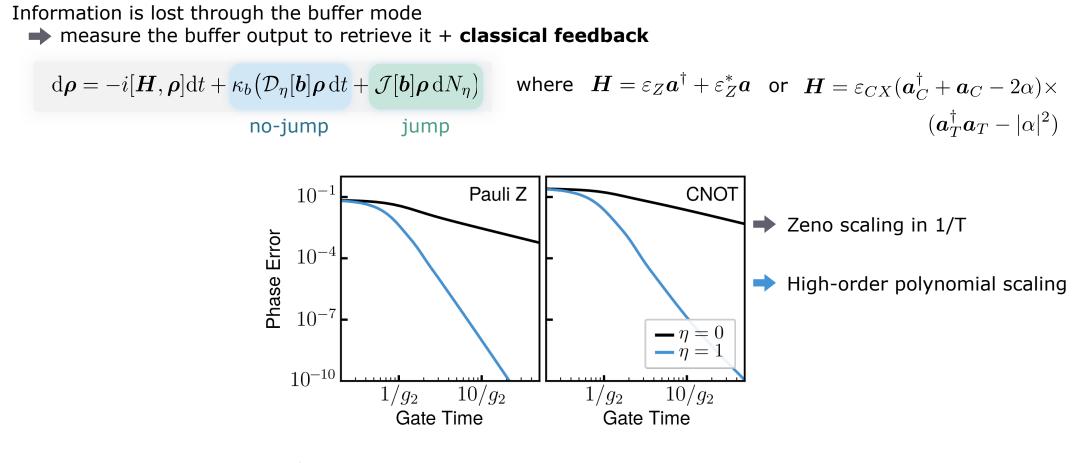


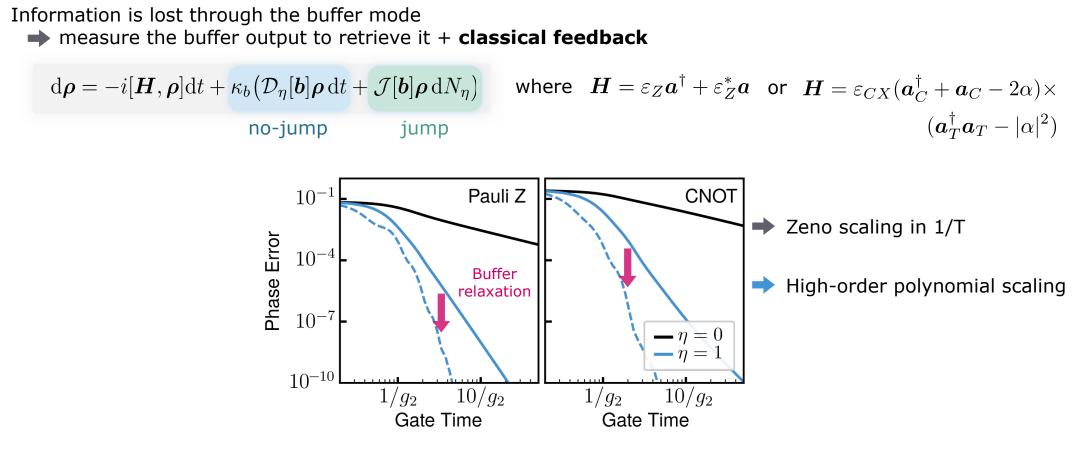
Information is lost through the buffer mode → measure the buffer output to retrieve it + **classical feedback**

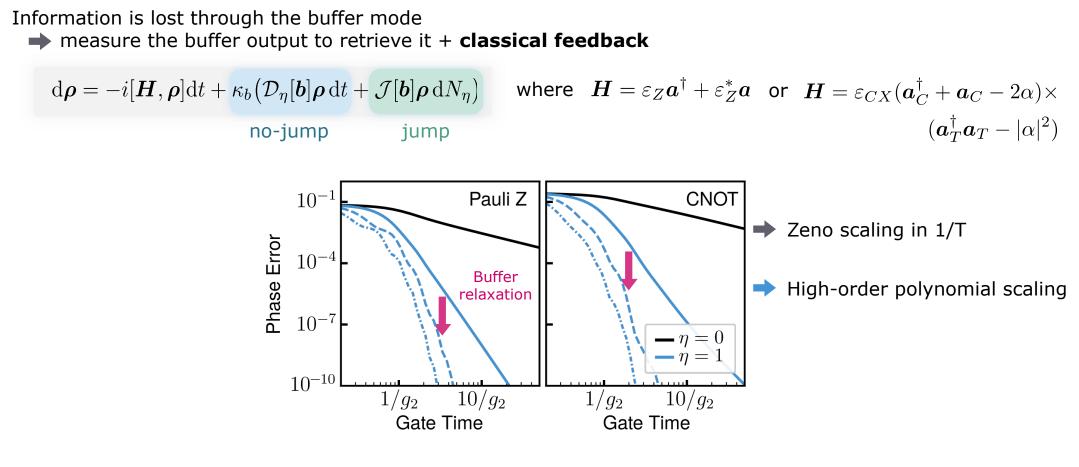
$$d\boldsymbol{\rho} = -i[\boldsymbol{H}, \boldsymbol{\rho}]dt + \kappa_b \left(\mathcal{D}_{\eta}[\boldsymbol{b}]\boldsymbol{\rho} dt + \mathcal{J}[\boldsymbol{b}]\boldsymbol{\rho} dN_{\eta} \right) \quad \text{where} \quad \boldsymbol{H} = \varepsilon_Z \boldsymbol{a}^{\dagger} + \varepsilon_Z^* \boldsymbol{a} \quad \text{or} \quad \boldsymbol{H} = \varepsilon_{CX} (\boldsymbol{a}_C^{\dagger} + \boldsymbol{a}_C - 2\alpha) \times (\boldsymbol{a}_T^{\dagger} \boldsymbol{a}_T - |\alpha|^2)$$

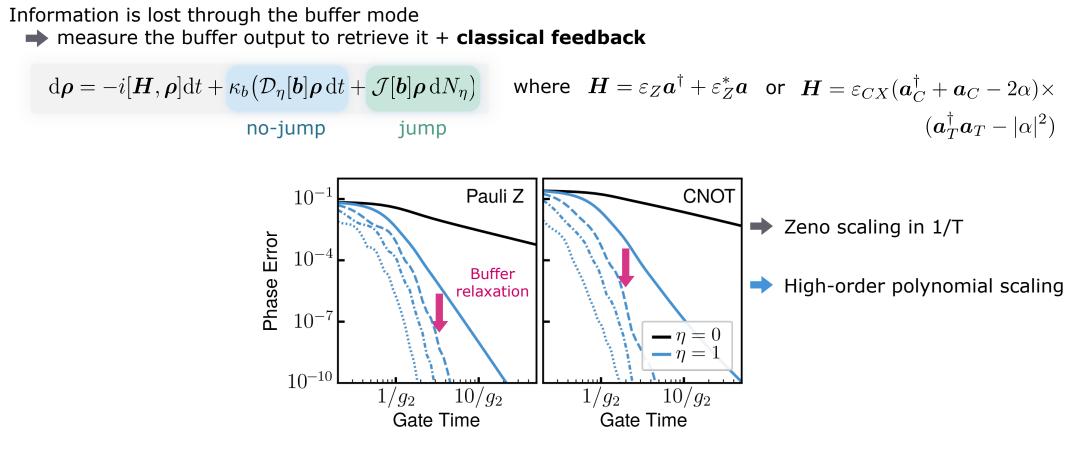


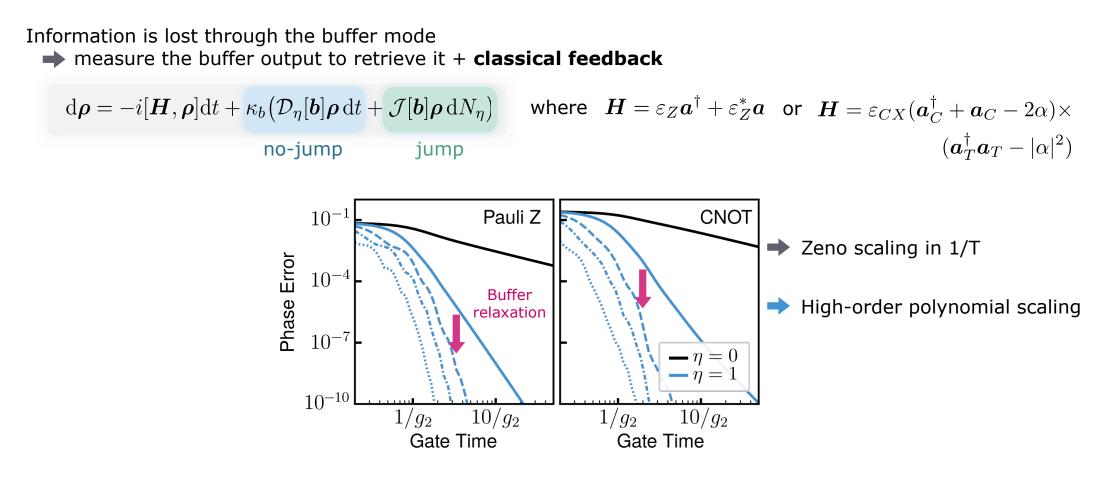




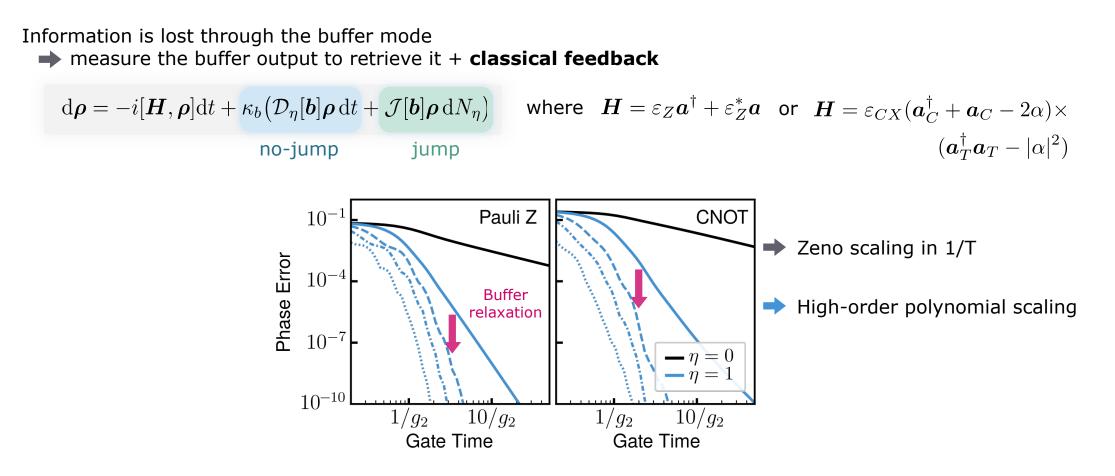








- Detection of buffer photons \rightarrow Erasure errors or classical feedback
- Can apply N gates with a single relaxation per QEC cycle



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- Can apply N gates with a single relaxation per QEC cycle
- Gate fidelity limited by detection efficiency **Autonomous feedback**

Gautier et al., arXiv (2023)

Information is lost through the buffer mode Feedback buffer detection autonomously

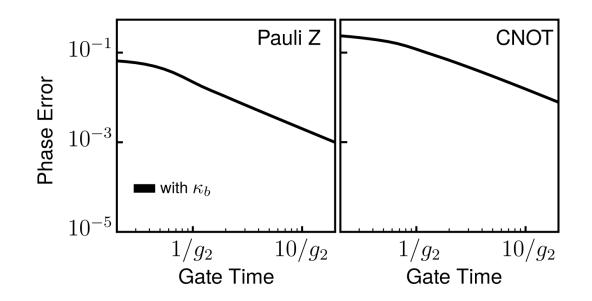
 $\frac{\mathrm{d}\boldsymbol{\rho}}{\mathrm{d}t} = -i[\boldsymbol{H}_{AB} + \boldsymbol{H}_{Z}, \boldsymbol{\rho}] + \frac{\kappa_{ab}\mathcal{D}[\boldsymbol{ab}]\boldsymbol{\rho}}{\kappa_{ab}\mathcal{D}[\boldsymbol{ab}]\boldsymbol{\rho}}$

➡ Correlate buffer photon losses with cat parity-swaps

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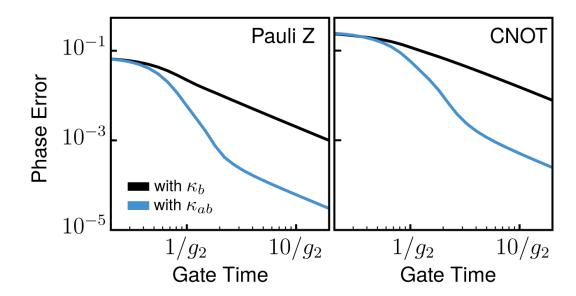
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Correlate buffer photon losses with cat parity-swaps

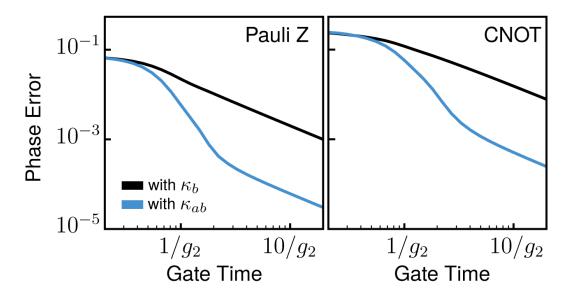


• **x50** gate fidelity improvement (limited by second-order effects), independent of $|lpha|^2$

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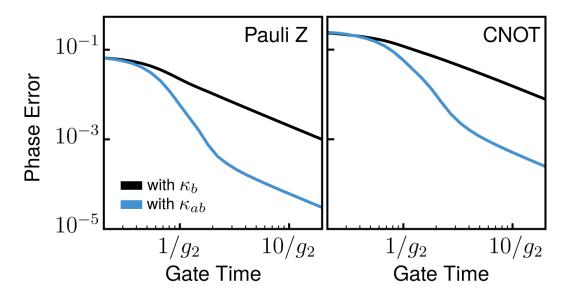


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Correlate buffer photon losses with cat parity-swaps

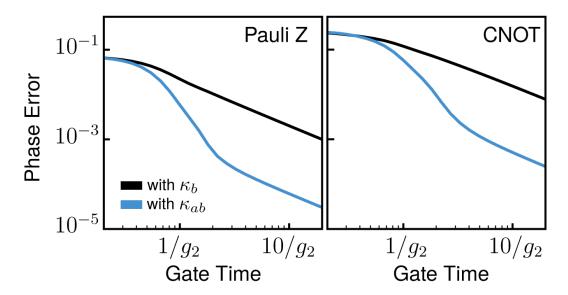


- **x50** gate fidelity improvement (limited by second-order effects), independent of $|lpha|^2$
- Generalizable to any CⁿX gate with no additional experimental overhead
- Autonomous correction of cavity losses with **squeezed** cats

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Correlate buffer photon losses with cat parity-swaps



- **x50** gate fidelity improvement (limited by second-order effects), independent of $|lpha|^2$
- Generalizable to any CⁿX gate with no additional experimental overhead
- Autonomous correction of cavity losses with **squeezed** cats
- Can tune dissipation in situ

Gautier et al., arXiv (2023); Xu, Jiang et al., arXiv (2022)

Designing High-Fidelity Gates for Dissipative Cat Qubits



- Buffer mode useful for more than dissipation engineering
- Information retrieval through photodetection: interesting conceptually but limited in practice
- Devised a scheme for autonomous correction of gate errors
- Check out arXiv:2303.00760 for more details

